

Chapter 13

Elementary Active Membranes Have the Power of Counting

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ABSTRACT

P systems with active membranes have the ability of solving computationally hard problems. In this paper, the authors prove that uniform families of P systems with active membranes operating in polynomial time can solve the whole class of PP decision problems, without using nonelementary membrane division or dissolution rules. This result also holds for families having a stricter uniformity condition than the usual one.

1. INTRODUCTION

P systems with active membranes (Păun, 2001) are known to solve computationally hard problems in polynomial time by trading space for time: an exponential number of membranes are created in polynomial time by using division rules, and then massive parallelism is exploited, e.g., to explore the whole solution space of an NP-complete problem in parallel.

When we allow nonelementary division rules, i.e., rules that can be applied to membranes containing further membranes, even PSPACE-

complete problems become solvable in polynomial time (Sosík, 2003; Alhazov, Martín-Vide, & Pan, 2003). The general idea is that nonelementary division allows us to construct a binary tree-shaped membrane structure, isomorphic to the parse tree of the formula resulting from the expansion of universal and existential quantifiers into conjunctions and disjunctions, according to the equivalences

$$\begin{aligned}\forall x \varphi(x) &\Leftrightarrow \varphi(0) \wedge \varphi(1) \\ \exists x \varphi(x) &\Leftrightarrow \varphi(0) \vee \varphi(1)\end{aligned}$$

We also know that no problem outside PSPACE can be solved in polynomial time, as this is also an upper bound (Sosík & Rodríguez-Patón, 2007): in symbols, we have $\text{PMC}_{\text{AM}} = \text{PSPACE}$.

On the other hand, when no division at all is allowed the resulting P systems can be shown to be no more powerful than polynomial-time Turing machines (Zandron, Ferretti, & Mauri, 2001).

The “intermediate” case, when the only membranes that can divide are elementary (i.e., leaves of the tree corresponding to the membrane structure), is possibly the most interesting one. The exponential number of membranes that may be created cannot be structured into a binary tree: hence, the algorithm above can only be applied to formulae having just one kind of quantifier. This is enough to solve the SAT problem (Zandron et al., 2001) and its complement (which are respectively NP- and coNP-complete), where only existentially (resp., universally) quantified variables are allowed.¹ However, the corresponding complexity class $\text{PMC}_{\text{AM}(-n)}$ still lacks a characterisation in terms of Turing machines. Alhazov et al. (2009) have shown how PP- and #P-complete problems can be solved without nonelementary division, but their result is not directly related to the class $\text{PMC}_{\text{AM}(-n)}$, as it requires some form of post-processing or the use of non-standard rules. In this paper, we improve the previous NP \cup coNP lower bound to PP within the standard framework of active membranes. This is an improved version of the paper “P systems with active membranes: Beyond NP and coNP” presented by the authors at the Eleventh International Conference on Membrane Computing (Porreca, Leporati, Mauri, & Zandron, 2010).

2. PRELIMINARIES

We use P systems with restricted elementary active membranes, which are defined as follows.

Definition 1: A P system with restricted elementary active membranes of initial degree $d \geq 1$ is a tuple $\Pi = (\Gamma, \Lambda, \mu, w_1, \dots, w_d, R)$, where:

- Γ is a finite alphabet of symbols (the objects);
- Λ is a finite set of labels for the membranes;
- μ is a membrane structure (i.e., a rooted *unordered tree*) consisting of d membranes enumerated by $1, \dots, d$; furthermore, each membrane is labeled by an element of Λ , not necessarily in a one-to-one way;
- w_1, \dots, w_d are strings over Γ , describing the initial multisets of objects placed in the d regions of μ ;
- R is a finite set of rules.

Each membrane possesses, besides its label and position in μ , another attribute called *electrical charge* (or polarization), which can be either neutral (0), positive (+) or negative (−) and is always neutral before the beginning of the computation.

The rules are of the following kinds:

Object evolution rules, of the form $[a \rightarrow w]_h^\alpha$

They can be applied inside a membrane labeled by h , having charge α and containing an occurrence of the object a ; the object a is rewritten into the multiset w (i.e., a is removed from the multiset in h and replaced by every object in w).

Send-in communication rules, of the form $a \left[\right]_h^\alpha \rightarrow \left[b \right]_h^\beta$

They can be applied to a membrane labeled by h , having charge α and such that the external region contains an occurrence of the object a ; the object a is sent into h becoming b and, simultaneously, the charge of h is changed to β .

Send-out communication rules, of the form $\left[a \right]_h^\alpha \rightarrow \left[\right]_h^\beta b$

They can be applied to a membrane labeled by h , having charge α and containing an occurrence of the object a ; the object a is sent out from h to the outside region becoming b and, simultaneously, the charge of h is changed to β .

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