Chapter 10

Using Swarm Intelligence for Optimization of Parameters in Approximations of Fractional-Order Operators

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ABSTRACT

This chapter applies Particle Swarm Optimization (PSO) to rational approximation of fractional order differential or integral operators. These operators are the building blocks of Fractional Order Controllers, that often can improve performance and robustness of control loops. However, the implementation of fractional order operators requires a rational approximation specified by a transfer function, i.e. by a set of zeros and poles. Since the quality of the approximation in the frequency domain can be measured by the linearity of the Bode magnitude plot and by the “flatness” of the Bode phase plot in a given frequency range, the zeros and poles must be properly set. Namely, they must guarantee stability and minimum-phase properties, while enforcing zero-pole interlacing. Hence, the PSO must satisfy these requirements in optimizing the zero-pole location. Finally, to enlighten the crucial role of the zero-pole distribution, the outputs of the PSO optimization are compared with the results of classical schemes. The comparison shows that the PSO algorithm improves the quality of the approximation, especially in the Bode phase plot.

DOI: 10.4018/978-1-4666-2666-9.ch010
INTRODUCTION: MATHEMATICAL BACKGROUND AND LITERATURE REVIEW

Fractional Calculus (FC) is a topic older than three centuries. Namely, its birth can be dated back to an exchange of letters and ideas between Leibniz and marquis de L’Hôpital in 1695. In this correspondence, de L’Hôpital asked to Leibniz what could be the value and meaning of a non-integer (fractional) order derivative \( \frac{d^\nu}{dx^\nu} \), more specifically with \( \nu = 0.5 \). This could extend the classical integer order derivative \( \frac{d^n}{dx^n} \), with \( n \in \mathbb{N} \), to a more general case, in which the order of differentiation \( \nu \) could be a fractional number. Considering \( n \in \mathbb{Z} \) and \( \nu < 0 \), an integer order integral could be extended to a fractional order one. Leibniz gave the result and answered on September 30, 1695, that “It will lead to an apparent paradox, from which one day useful consequences will be drawn”.

Since then many mathematicians and scientists (Euler, Abel, Lacroix, Fourier, Lagrange, Laplace, Riemann, Liouville, Kellang, Grünwald, Letnikov, Caputo, etc.) have formulated and investigated formal properties of non-integer order differentiation and integration. As an example, Heaviside said that “there is a universe of mathematics lying in between the complete differentiations and integrations”. At his time, he faced the problem of a rigorous justification for the square root operation of a partial differentiation operator \( p \). After algebraic manipulations, he obtained \( p^\alpha \), with \( \alpha \) non-integer number. Moreover, he first discovered the value of \( \sqrt{p} = p^{0.5} \) as an experimental solution of a heat flow problem, by applying classical methods with innovative computations for his time.

The idea of fractional derivative or integral can be described in different ways. The most popular definitions are due to Riemann-Liouville, to Grünwald-Letnikov, and to Caputo. The Riemann-Liouville basic definition of fractional order integral generalizes the repeated integration in the Cauchy formula:

\[
R_L a D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t - \tau)^{\alpha - 1} f(\tau) \, d\tau
\]

(1)

where \( \alpha \in \mathbb{R} \), with \( \alpha > 0 \), is the non-integer order of integration with respect to \( t \) and starting point \( a \), and \( \Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt \) is the Euler gamma function. Then, the Riemann-Liouville definition of fractional order derivative can be obtained as follows:

\[
R_L a D_t^\nu f(t) = \frac{d^n}{dt^n} \left\{ R_L a D_t^{-n+\nu} f(t) \right\} = \frac{1}{\Gamma(n - \alpha)} \frac{d^n}{dt^n} \left\{ \int_a^t (t - \tau)^{n-\alpha - 1} f(\tau) d\tau \right\}
\]

(2)

where \( \alpha \in \mathbb{R} \), with \( \alpha > 0 \) and \( n - 1 < \alpha < n \), \( n \in \mathbb{N} \), is now the non-integer order of differentiation. The Laplace transform of the fractional order integral operator, provided that \( \mathcal{L}\{f(t)\} = F(s) \), gives: \( \mathcal{L}\left\{ R_L a D_t^\nu f(t) \right\} = s^{-\alpha} F(s) \), that is analogous to the standard transform of integer order integral operator. Whereas, the Laplace transform of the fractional order derivative operator leads to:

\[
\mathcal{L}\left\{ R_L a D_t^\nu f(t) \right\} = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k \left. R_L a D_t^{n-k-1} f(t) \right|_{t=0}
\]

(3)

However, evaluation of Equation 3 requires initial values of the fractional order derivatives that are of difficult interpretation and measurement.

The Caputo definition avoids physical interpretation of initial conditions in Laplace transform of the fractional order derivative operator by writing:
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