Chapter 19
Nonparametric Estimation of Nonlinear Dynamics by Local Linear Approximation

Isao Shoji
University of Tsukuba, Japan

ABSTRACT

This chapter discusses nonparametric estimation of nonlinear dynamical system models by a method of metric-based local linear approximation. By specifying a metric such as the standard metric or the square metric on the Euclidean space and a weighting function based on such as the exponential function or the cut-off function, it is possible to estimate values of an unknown vector field from experimental data. It can be shown the local linear fitting with the Gaussian kernel, or the local polynomial modeling of degree one, is included in the class of the proposed method. In addition, conducting simulation studies for estimating random oscillations, the chapter shows the method numerically works well.

INTRODUCTION

Recent rapid progresses in information technology enable us to use very high frequency data such as sampled at every minute or second. So, when modeling time series with such high frequencies, we could apply continuous time stochastic processes formulated by stochastic differential equations (SDEs). Actually, there are wide varieties of their applications to economics, finance, neurosciece, and electric engineering. Particularly, their theoretical and empirical applications to finance are quite extensive; see Campbell et al. (1997) for example. Like conventional regression models in discrete time, we have to specify two coefficients of a SDE, called the drift coefficient f and the diffusion coefficient σ as follows:

$$dX_t = f(X_t)dt + \sigma(X_t)dB_t, \tag{1}$$

where X_t stands for the process of interest and B_t for the standard Brownian motion.

These coefficients are formulated by specific functions according to some theories or simply due to tractability for manipulation. In highly complex systems, however, the functions are presumably

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nonlinear while almost always we are expected to have no theory about the system under consideration and thus have no exact knowledge about its functional form. Needless to say, specification of these coefficients is needed for good modeling, and especially specification of the drift coefficient is crucial because it determines the mean trajectory of the process. If misspecified, such a bad model leads to entire lack of fit into data so that any statistical inference based on the model should be quite suspicious and its forecasting power would be poor. To avoid the misspecification, an expected solution is to find a method to specify these coefficients not by a priori theory but by efficiently gathering information from data.

To this end, we might use nonparametric models instead of parametric ones because nonparametric models are thought to have less risk of misspecification. The studies of estimating an unknown function nonparametrically have been continued since 1990's or before. For example, Casdagli (1991) discusses various applications of the nearest neighborhood method by which the forecasting functions in chaos systems are estimated from time series. n our approach, however, we want to choose another method by which we estimate a local structure of the vector field $f(x)$, more specifically the tangent space of the curve or surface implied by $f(x)$.

To detect the local structure, we explain two methods, that is the local linear fitting and the metric-based local linear approximation. The local linear fitting is known to be the local polynomial model of degree 1, which is extensively addressed by Fan and Gijbels (1996), Fan and Yao (2005), Gyorfi et al (2010), Li and Racine (2007), and Wasserman (2007) for example. The metric-based local linear approximation is proposed by Shoji (2011). Both of the methods are basically to estimate the tangent plane at a point nonparametrically from data. But, the way of handling the locality is different from each other; the local linear fitting uses so-called kernel functions to characterize the locality, and on the other hand, the metric-based local linear approximation uses the typical metrics on the Euclidean space. Regardless of the difference, we will show the latter contains the former. In other words, the metric-based local linear approximation is considered to be a generalization of the local linear fitting.

The organization of this chapter is as follows. In the section below, we explain the local linear fitting with the Gaussian kernel and then, in a further section, the metric-based local linear approximation. Later in the chapter, the local linear fitting is shown to be contained by the metric-based local linear approximation. We will also explain how to select a neighborhood, and we carry out simulation studies to compare the performance of estimation by the metric-based local linear approximation with the local linear fitting. At the end of the chapter, we give the concluding remarks.

LOCAL LINEAR FITTING

The local linear model is a restricted version of the local polynomial model of degree 1, say p=1 below. Basically in line with Fan and Gijbels (1996), Fan and Yao (2005), Gyorfi et al (2010), Li and Racine (2007), and Wasserman (2007), we try to explain the local polynomial model including higher order. For notational simplicity, we focus on one-dimensional processes in this section.

The local polynomial model is a method to fit polynomial functions locally into an unknown function. Each polynomial function may be different from one after another for every local domain, or local neighborhood, specified by the so-called kernel functions. So, the method is not a simple global polynomial model.

In the first place, suppose Y is expressed as a regression model of X defined by,

$$Y = f(X) + \varepsilon_t, \tag{2}$$

where $f(x)$ is assumed to be a smooth, possibly nonlinear, function but unknown.