

Chapter 2

On Dynamical Behaviors and Chaos Control of the Fractional–Order Financial System

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ABSTRACT

In this work, the authors investigate the stability conditions in a fractional-order financial system using the fractional Routh-Hurwitz criteria. According to the qualitative theory, the existence and uniqueness of solutions for a class of commensurate fractional-order financial systems are investigated. A necessary condition for this system to remain chaotic is obtained. It is found that chaos exists in this system with order less than three. Furthermore, the fractional Routh-Hurwitz conditions are used to control chaos in the proposed fractional-order system to its equilibria via linear feedback control method. Then, it is shown that the fractional-order financial system is controllable just in the fractional-order case when using a specific choice of linear controllers. Numerical simulations are used to verify the analytical results.

INTRODUCTION

The story of fractional calculus has been initiated since 300 years (Podlubny, 1999). Facts show that many systems can be elegantly described with the help of fractional derivatives in inter-

disciplinary fields, for example, electromagnetic waves (Heaviside, 1971), visco-elastic systems (Bagley & Calico, 1991), quantum evolution of complex systems (Kusnezov et al, 1999) and diffusion waves (El-Sayed, 1996). Furthermore, applications of fractional calculus have been reported in many areas such as physics (Hilfer,

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2000), engineering (Sun et al, 1984), quantitative finance (Laskin, 2000), social sciences (Ahmad & El-Khazali, 2007), and mathematical biology (Ahmed & Elgazzar, 2007; El-Sayed et al, 2007).

There are several definitions of the fractional derivatives. Here, we use the Caputo-type fractional derivative (Caputo, 1967) which is given as follows:

$$D^\alpha f(t) = \frac{d^\alpha}{dt^\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, & m-1 \leq \alpha < m \\ \frac{d^m}{dt^m} f(t), & \alpha = m, \end{cases} \quad (1.1)$$

where m is the first integer which is not less than α , i.e. $m-1 \leq \alpha < m$ and $\Gamma(\cdot)$ is the well-known Euler's gamma function:

$$\Gamma(\theta) = \int_0^\infty t^{\theta-1} e^{-t} dt, \quad (1.2)$$

The operator D^α is called "Caputo differential operator of order α ".

Chaos is an important dynamical phenomenon which has been extensively studied and developed by scientists since the work of Lorenz (Lorenz, 1963). A chaotic system has complex dynamical behaviors such as the unpredictability of the long-term future behavior and irregularity. According to the Poincaré-Bendixon theorem, chaos in fractional-order autonomous systems can occur for orders less than three and this can not happen in their integer-order counterparts. Chaos has important applications in the fields of industry and communications like chaos control (Ott et al, 1990) and chaos synchronization (Pecora & Carroll, 1990; Agiza & Matouk, 2006; Matouk & Agiza, 2008; Matouk, 2009a; Hegazi & Matouk, 2011).

On the other hand, there are practical situations where one wishes to avoid or control chaos so as to

improve the performance of a dynamical system, owing to the wide scale of observing chaos (almost all fields of life such as chemistry, economy, biology, physics...etc), chaos sometimes is undesirable and may lead to disaster; for example, increased drag in flow systems, erratic fibrillations of heart beating, extreme weather patterns and complicated circuit oscillations. Thus, our objective is to avoid, eliminate such behaviors. Recently, suitable and efficient methods have been developed for controlling chaos. One of the most efficient ways for controlling chaos is linear feedback control technique (Matouk, 2008). This technique has also been successfully applied to achieve chaos control in some chaotic fractional-order systems (Matouk, 2009b; Matouk, 2010; Matouk, 2011).

THE COMMENSURATE FRACTIONAL-ORDER FINANCIAL SYSTEM

The integer-order financial system is investigated in (Zhao et al, 2011):

$$\begin{aligned} dx / dt &= z + (y - \mu)x, \\ dy / dt &= 1 - \beta y - x^2, \\ dz / dt &= -x - \nu z, \end{aligned} \quad (2.1)$$

where x denotes the interest rate, y denotes the investment demand, and z denotes the price index.

The factors that influence changes in x mainly come from two aspects: first, contradictions from the investment market, i.e., the surplus between investment and savings, and second, structural adjustment from good prices. The changing rate of y is in proportion to the rate of investment, and in proportion to an inversion with the cost of investment and interest rates. Changes in z , on the one hand, are controlled by a contradiction between supply and demand in commercial markets, and on the other hand, are influenced by inflation rates. It should be noted

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