# Chapter 10
# Recurrence Indicators for the Estimation of Characteristic Size and Frequency of Spatial Patterns

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## ABSTRACT

*In this chapter, the authors propose a method for the estimation of the characteristic size and frequency of the typical structure in systems showing two dimensional spatial patterns. In particular, they use several indicators caught from the nonlinear framework for identifying the small and large scales of the systems. The indicators are applied to the images corresponding to the instantaneous realization of the system. The method assumes that it is possible to capture the main system's properties from the distribution of the recurring patterns in the image and does not require the knowledge of the dynamical system generating the patterns neither the application of any image segmentation method.*

## 1. INTRODUCTION

Recurrence Plot (RP) and Recurrence Quantification Analysis (RQA) have been widely used as visual and quantitative tools for the analysis of nonlinear time series. In fact, they give a visual insight of the system dynamics when complex behaviors, such as deterministic chaos, are observed.

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In recent years, RPs found a wide range of applications in the time series analysis of nonstationary phenomena, such as biological systems (P. Kaluzny, 1993, N. Thomasson, 2001, N. Marwan, 2002, J. P. Zbilut, 2004), speech analysis (L. Matassini, 2002, A. Facchini, 2005), financial time series (F. Strozzi, 2007), and earth sciences (N. Marwan, 2001, N. Marwan, 2003). The popularity of RPs lies in the fact that their structures are visually appealing, and that they allow for the investigation of high dimensional dynamics by means of a simple two-dimensional plot.

Furthermore, Recurrence Plots and Recurrence Quantification Analysis have been used for the detection of structural changes in the dynamics of complex nonlinear systems. For example, the relationship between the RP and sequences of bifurcations in the logistic equation has been discussed by (L.L. Trulla, 1996) for the detection of bifurcations without making any a priori assumption on the underlying equations of motion. (J. Gao, 2000) cope with the problem of identifying true bifurcation sequences and the causes of possibly false bifurcation points. They also give indications on how to choose suitable embedding parameters so that the bifurcation detection may still work when the signal is heavily corrupted by noise.

Recently, several extensions of RP and RQA to spatio-temporal systems have been proposed. (D.B. Vasconcelos, 2006) give a first definition of Spatial Recurrence Plot (SRP) and show that SRP allows one to detect some spatial patterns, like the roughness in metallic materials. They finally argue that SRP works well for recognizing other more general kinds of spatial structures. The extension of the RP to $d$-dimensional data sets, i.e. the Generalized Recurrence Plot (GRP) and Generalized Recurrence Quantification Analysis (GRQA), has been proposed by (N. Marwan, 2007(a)) with an application to the analysis of the bone structure from Computer Tomography images.

Recently, the GRP has been exploited for the characterization of spatially distributed systems (A. Facchini, 2009(a)) and investigation of pattern formation in chemical systems (A. Facchini, 2009(b)). A further application of this method to the analysis of and detection of bifurcations in spatially distributed systems has been recently published in (C. Mocenni, 2010), while analogies and differences between RP and GRP for spatial domains are discussed in (C. Mocenni, 2011).

This letter investigates the relationship between the recurrence measures and the pattern size and distribution in two dimensional domains. In section 2 the Recurrence Plots and the Recurrence Quantification analysis are introduced. Section 3 describes the experimental set-up and states the main results. Finally, concluding remarks and future work are reported in section 4.

## 2. RECURRENCE ANALYSIS OF COMPLEX SPATIALLY DISTRIBUTED SYSTEMS

In order to achieve a better understanding of the spatial extension, a short introduction on RP is given in the following, for a deeper treatment the reader is referred to (N. Marwan, 2007).

### 2.1 Time Series Analysis Based on Recurrence Plots

Let $x_t$ be a time series and $S_m \subseteq R^m$ the associated $m$-dimensional embedded state space, reconstructed using the Takens delay Theorem ((F. Takens, 1981)). The Recurrence Plot is essentially a two dimensional binary diagram indicating the recurrences that occur in the space $S_m$ within a fixed threshold ε at different times $i,j$. The RP is easily expressed as a two dimensional square matrix $R = \{r_{ij} : i, j = 1, \dots, N\}$, with:

$$r_{ij} = \tau(\varepsilon - \left\|\vec{x}_i - \vec{x}_j\right\|) \tag{1}$$

where $x_i, x_j \in S_m$, $i,j=1,\dots,N$, $N$ is the number of the measured states $\vec{x}_i$, $\tau(\cdot)$ is the step function, and $\|\cdot\|$ is a norm. In the graphical representation, each non-zero entry of the RP matrix **R** is marked by a black dot in the position $(i,j)$. Since any state is recurrent with itself, the RP matrix **R** fulfills $r_{i,i} = 1$, i.e. the RP contains the diagonal, called *Line of Identity* (LOI).

The structures that can be identified in the RP give insightsnabout the underlying dynamics are:

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