

## Chapter 3

# The Node-to-Node Graph Matching Algorithm Schema

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### ABSTRACT

*Many graph matching algorithms follow the approach of node-similarity measurement, that is, matching graphs by means of comparing the corresponding pairs of nodes of the graphs. Based on this idea, the authors propose a high-level schema for node-to-node graph matching, namely N2N graph matching algorithm schema. The chapter shows that such a N2N graph matching algorithm schema is versatile enough to subsume most of the representative node-to-node based graph matching algorithms. It is also shown that improved algorithms can be derived from this N2N graph matching schema, compared with various corresponding algorithms. In addition, the authors point out the limitation and constraints of the propose algorithm schema and suggest some possible treatments.*

### INTRODUCTION

Graphs are a powerful and versatile tool used for the description of structural objects which has been widely used in mathematics, computer science, artificial intelligence, biology, geography, or even politics, for representing structural objects and concepts. By graph representation, the task of calculating the similarity degree between two

objects can be simply transferred into the problem of matching the corresponding pair of graphs.

Various algorithms for graph matching problems have been developed, which, according to Gold and Rangarajan (1996), can be classified into two categories: (1) search-based methods which rely on possible and impossible pairings between vertices; and (2) optimization-based methods which formulate the graph matching

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problem as an optimization problem. Generally speaking, explicit search methods directly search the optimal match among permutation space (or permutation matrices space). Since the size of the search spaces increase exponentially according to the graph size, different kinds of heuristic techniques are developed to reduce the search space to a smaller acceptable size. Implicit search methods do not search for the optimal match in permutation space; instead, the permutation space is transferred into some other continuous real number space or mixed 0-1 and real number space and meanwhile the graph matching problems is also represented as an optimization among the continuous or mixed space.

Actually, there are other kinds of graph matching algorithms which do not use any search; instead, they simply explore some kind of node similarity between nodes of graph pairs, and get the optimal solutions by matching those similar nodes. In this paper, those matching algorithms shall be unified as the N2N graph matching algorithm schema, and detailed studied including the formulism of N2N graph matching algorithm schema, the examples of such schema, comparisons of some N2N graph matching algorithms, the limitation of such the N2N graph matching algorithm schema, and possible ways to improve it.

## BACKGROUND

In 1987, Umeyama proposed an eigen-decomposition based graph matching algorithm (EDGM) for matching both undirected and directed weighted graphs. The EDGM algorithm is critical examined in (Zhao et al., 2007) that EDGM algorithm only works for graphs with single eigenvalues, and improved as meta-basis based graph matching algorithm MBGM, which can be applied for more general cases.

In 1991, Almohamad presented a symmetric polynomial transform based graph matching algorithm (SPGM), where the node similarity of

two graphs is constructed by the coefficient of the polynomial transform of the weights of the edges.

In 1999, Kleinberg proposed a hubs and authorities graph matching algorithm (HAGM) for internet searching, where the node similarity is based on the idea that two nodes are similarity if their adjacent nodes are similar. An iterative algorithm is provided to calculate such node similarity. This algorithm has been revised in (Zager & Verghese, 2008).

In 2003, van Wyk & van Wyk presented several Kronecker product successive projection based graph matching algorithms. The graph matching problem is transferred into the Kronecker Product Graph Matching formulation, based on which several approaches are derived, such as the least squares Kronecker product graph matching (LSKPGM) algorithm, the interpolator-based Kronecker product graph matching (IBKPGM) algorithm, the gradient-based Kronecker product graph matching (GBKPGM) algorithm and the orthonormal kernel Kronecker product graph matching (OKKPGM) algorithm.

Although these methods are derived from different theories, they are using the same idea to matching graphs by node similarity. These methods are mostly only applicable for certain kinds of graphs, but they can be easily implemented, analyzed and improved. In addition, most of the node similarity based graph matching algorithms have low computational complexities and are consequently applicable to large size graphs.

## MAIN FOCUS OF THE CHAPTER

Some basic notions have to be defined before the discussion.

- **Definition 1:** A permutation  $p$  is an one-to-one function from  $\{1, 2, \dots, n\}$  to itself. The set of all permutations of  $n$  elements is denoted as  $SG(n)$ .
- **Definition 2:** A permutation matrix  $P$  is a  $n$ -by- $n$  matrix such that

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