Chapter XV
Attractors and Energy Spectrum of Neural Structures Based on the Model of the Quantum Harmonic Oscillator

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ABSTRACT

Neural computation based on principles of quantum mechanics can provide improved models of memory processes and brain functioning and is of primary importance for the realization of quantum computing machines. To this end, this chapter studies neural structures with weights that follow the model of the quantum harmonic oscillator. The proposed neural networks have stochastic weights which are calculated from the solution of Schrödinger's equation under the assumption of a parabolic (harmonic) potential. These weights correspond to diffusing particles, which interact with each other as the theory of Brownian motion (Wiener process) predicts. The learning of the stochastic weights (convergence of the diffusing particles to an equilibrium) is analyzed. In the case of associative memories the proposed neural model results in an exponential increase of patterns storage capacity (number of attractors). It is also shown that conventional neural networks and learning algorithms based on error gradient can be conceived as a subset of the proposed quantum neural structures. Thus, the complementarity between classical and quantum physics is also validated in the field of neural computation.

INTRODUCTION

In this chapter, neural structures with weights that follow the model of the quantum harmonic oscillator will be studied. Connectionist structures which are compatible with the theory of quantum mechanics and demonstrate the particle-wave nature of information, have been analyzed in (Hagan et. al., 2002), (Perus et. al., 2004), (Rigatos & Tzafestas, 2007a). This chapter extends results on the compatibility of neural structures with quantum mechanics principles, presented in (Rigatos &. Tzafestas, 2002), (Rigatos & Tzafestas, 2006a).

It is assumed that the neural weights are stochastic variables which correspond to diffusing particles, and interact to each other as the theory of Brownian motion predicts. Brownian motion is the analogous of the quantum harmonic oscillator (Q.H.O.), i.e. of Schrödinger's equation under harmonic (parabolic) potential. It will be shown that the update of the stochastic weights is described by Langevin's stochastic differential equation which is a generalization of conventional gradient algorithms. It will also be shown that weights following the Q.H.O. model give to associate memories significant properties: (i) the learning of the weights is a Wiener process, and (ii) the number of attractors increases exponentially comparing to conventional associative memories.

The structure of the chapter is as follows: In Section "*Equivalence between Schrödinger's equation and a diffusion process*", an analysis of the diffusive motion of particles (stochastic weights) is given and the background that relates Schrödinger's equation with diffusive motion is analyzed. It is shown that Schrödinger's equation with harmonic potential is equivalent to a stationary diffusion, and that the motion of the diffusing particles is described by Langevin's equation. In section "*Interacting diffusing particles as a model of neural networks*", a neural model based on interacting diffusing particles, is proposed. In Section "*Compatibility with principles of quantum mechanics*", it is shown that using directly Schrödinger's equation in place of the previously analyzed stochastic processes, the stochastic weights of the neural network can be described by a probability density function which stems again from the model of the quantum harmonic oscillator. In section "*Attractors in Associative Memories based on the Q.H.O. model*", it is shown that the Q.H.O. model in associative memories results in exponential increase of the number of attractors while the the update of the weights stands for a Wiener process. In section "*Spectral analysis of associative memories that follow the QHO model*", spectral analysis shows that the stochastic weights satisfy a relation analogous to the principle of uncertainty. In section "*Simulation tests*", simulation results are presented about the convergence of Brownian weights to attractors and about the exponential increase of the number of attractors in associative memories with Brownian weights. Finally, in Section "*Conclusions*", concluding remarks are stated.

BACKGROUND

A detailed analysis of the current status of research on neural networks and cognitive models based on the principles of quantum mechanics will be given. As it can be observed from the relevant bibliography the field is wide and with enormous potential. Without excluding other approaches on neural structures with quantum mechanical features, in this chapter three main research directions are distinguished: (1) Neural structures which use as activation functions the eigenstates of the quantum harmonic oscillator, (2) Neural structures with stochastic weights. Sub-topics in this area are (a) neural structures with stochastic weights which stem from the solution of Schrödinger's linear equation for constant or zero potential (b) stochastic neural networks which can be modelled as gene networks (3). Neural structures with stochastic weights which stem from the model of the quantum harmonic oscillator

1. Results on neural structures which use as activation functions the eigenstates of the quantum harmonic oscillator: Feed-forward neural networks (FNN) are the most popular neural architectures due to their structural flexibility, good representational capabilities, and availability of a large number of training algorithms. The hidden units in a FNN usually have the same activation functions and are usually selected as sigmoidal functions or gaussians. Feed-forward neural networks that use the eigenstates of the quantum harmonic oscillator (QHO) as basis functions have some interesting properties: (i) the basis functions are invariant under the Fourier transform,