

Chapter 16

Response Curves for Cellular Automata in One and Two Dimensions: An Example of Rigorous Calculations

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ABSTRACT

In this paper, the authors consider the problem of computing a response curve for binary cellular automata, that is, the curve describing the dependence of the density of ones after many iterations of the rule on the initial density of ones. The authors demonstrate how this problem could be approached using rule 130 as an example. For this rule, preimage sets of finite strings exhibit recognizable patterns; therefore, it is possible to compute both cardinalities of preimages of certain finite strings and probabilities of occurrence of these strings in a configuration obtained by iterating a random initial configuration n times. Response curves can be rigorously calculated in both one- and two-dimensional versions of CA rule 130. The authors also discuss a special case of totally disordered initial configurations, that is, random configurations where the density of ones and zeros are equal to $1/2$.

INTRODUCTION

Cellular automata (CA) can be viewed as computing devices, which take as an input some initial configuration. The CA rule is iterated a number of times starting from this configuration, resulting in

a final configuration, which constitutes the output of the computation. If the CA rule is complex, the above computation may be very difficult to characterize, let alone understand in detail. If one considers “simple” CA rules, however, one can say quite a lot about the process, as we shall see in what follows.

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In many practical problems, e.g., in mathematical modelling, one wants to know how a CA rule iterated over an initial configuration affects certain aggregate properties of the configuration, such as, for example, the density of ones. If we take a randomly generated initial configuration with a given density of ones, and iterate a given rule n times over this configuration, what is the density of ones in the resulting configuration? Using signal processing terminology, we want to know the “response curve”, density of the output as a function of the density of the input. Response curves appear in computational problems, and a classical example of such a problem in CA theory is the so-called density classification problem (DCP). If we denote the density of ones in the configuration at time n by c_n , the DCP asks us to find a rule for which $c_\infty = 1$ if $c_0 > 1/2$ and $c_\infty = 0$ if $c_0 < 1/2$ - that is, CA rule with density response curve in a form of a step function. Since it is known that such a rule does not exist (Land and Belew, 1995), one could ask a related general question: which response curves are possible in CA rules? Obviously, this problem is much more difficult than DCP, and not much is known about it. We propose to approach this problem from an opposite direction: given the CA rule, what can we say about its response curve? It turns out that in surprisingly many cases, the response curve can be calculated exactly, providing that preimage sets of finite strings under the CA rule exhibit recognizable patterns. We will demonstrate this technique using as an example elementary CA rule 130, in both one- and two-dimensional spaces.

BASIC DEFINITIONS

Let $\mathcal{N} = \{0,1\}$ be called a *symbol set*, and let $\mathcal{S}(\mathcal{N})$ be the set of all bisequences over \mathcal{N} , where by a bisequence we mean a function on Z

to \mathcal{N} . Throughout the remainder of this text the configuration space $\mathcal{S}(\mathcal{N}) = \{0,1\}^Z$ will be simply denoted by \mathcal{S} .

A *block of length n* is an ordered set $b_0 b_1 \dots b_{n-1}$, where $n \in N$, $b_i \in \mathcal{N}$. Let $n \in N$ and let \mathcal{B}_n denote the set of all blocks of length n over \mathcal{N} and \mathcal{B} be the set of all finite blocks over \mathcal{N} .

For $r \in N$, a mapping $f : \{0,1\}^{2r+1} \mapsto \{0,1\}$ will be called a *cellular automaton rule of radius r* . Alternatively, the function f can be considered as a mapping of \mathcal{B}_{2r+1} into $\mathcal{B}_0 = \mathcal{N} = \{0,1\}$.

Corresponding to f (also called a *local mapping*) we define a *global mapping* $F : \mathcal{S} \rightarrow \mathcal{S}$ such that $(F(s))_i = f(s_{i-r}, \dots, s_i, \dots, s_{i+r})$ for any $s \in \mathcal{S}$. The *composition of two rules* f, g can be now defined in terms of their corresponding global mappings F and G as $(F \circ G)(s) = F(G(s))$, where $s \in \mathcal{S}$.

A *block evolution operator* corresponding to f is a mapping $\mathbf{f} : \mathcal{B} \mapsto \mathcal{B}$ defined as follows. Let $r \in N$ be the radius of f , and let $a = a_0 a_1 \dots a_{n-1} \in \mathcal{B}_n$ where $n \geq 2r + 1 > 0$. Then

$$\mathbf{f}(a) = \{f(a_i, a_{i+1}, \dots, a_{i+2r})\}_{i=0}^{n-2r-1}. \quad (1)$$

Note that if $b \in \mathcal{B}_{2r+1}$ then $f(b) = \mathbf{f}(b)$.

We will consider the case of $\mathcal{N} = \{0,1\}$ and $r = 1$ rules, i.e., *elementary cellular automata*. In this case, when $b \in \mathcal{B}_3$, then $f(b) = \mathbf{f}(b)$. The set $\mathcal{B}_3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$ will be called the set of *basic blocks*.

The number of n -step preimages of the block b under the rule f is defined as the number of elements of the set $\mathbf{f}^{-n}(b)$. Given an elementary rule f , we will be especially interested in the number of n -step preimages of basic blocks under the rule f .

As mentioned in the introduction, we will use as an example rule 130 (using Wolfram’s numbering scheme) with local function defined as

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