

# Chapter 13

## Generalized Rough Logics with Rough Algebraic Semantics

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### ABSTRACT

*The collection of the rough set pairs  $\langle \text{lower approximation}, \text{upper approximation} \rangle$  of an approximation  $(U, R)$  can be made into a Stone algebra by defining two binary operators and one unary operator on the pairs. By introducing a more unary operator, one can get a regular double Stone algebra to describe the rough set pairs of an approximation space. Sequent calculi corresponding to the rough algebras, including rough Stone algebras, Stone algebras, rough double Stone algebras, and regular double Stone algebras are proposed in this paper. The sequent calculi are called rough Stone logic (RSL), Stone logic (SL), rough double Stone logic (RDSL), and double Stone Logic (DSL). The languages, axioms and rules are presented. The soundness and completeness of the logics are proved.*

### INTRODUCTION

Rough set theory was invented by Pawlak to account for the definability of a concept in terms of some elementary ones in an approximation space  $(U, R)$ , where  $U$  is a set, and  $R$  is an equivalence relation on  $U$ . It captures and formalizes the basic phenomenon of information granulation. The finer the granulation is, the more concepts are

definable in it. For those concepts not definable in an approximation space, the lower and upper approximations for them can be defined. These approximations construct a representation of the given concept in the approximation space.

Research on rough sets by algebraic method has gathered many researchers' attention such as (Iwinski, 1987; Pomykala et al., 1988; Gehrke, 1992; Comer, 1993; Lin et al., 1994; Pagliani, 1996; Banerjee et al., 1996; Banerjee, 1997; Jarvinen, 2002; Dai, 2004; Dai, et al., 2006(a), Dai

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et al., 2006(b); Dai, 2007; Dai, 2008). In (Lin et al., 1994), the researchers studied the approximations under the structure of set algebra. Iwinski paid more attention about the lattice properties of set algebra and proposed the method of defining of rough approximations (Iwinski, 1987). Based on atomic Boolean lattice, Jarvinen proposed a novel definition method of rough approximations (Jarvinen, 2002). Based on the molecules, Dai constructed a more general structure of rough approximations based on molecular lattices (Dai 2004). At the same time, researchers also study rough sets from the point of description of the rough set pairs i.e.  $\langle \text{lower approximation set, upper approximation set} \rangle$ . Pomykala and Pomykala laid a foundation for this field (Pomykala et al., 1988). They used Stone algebra to describe rough sets. They showed that collection of the rough set pairs of an approximation  $(U, R)$  can be made into a Stone algebra. By introducing another unary operator, Comer got a regular double Stone algebra to describe the rough set pairs of an approximation space (Comer, 1993). Pagliani adopted semi-simple Nelson algebra (Pagliani, 1996). Banerjee and Chakraborty defined pre-rough algebra which is more structured than topological quasi-Boolean algebra and used pre-rough algebra to describe rough sets (Banerjee et al., 1996). Some rough algebras with Brouwer-Zadeh lattices and 3-valued Lukasiewicz algebras were connected (Dai et al., 2006(a); Dai et al., 2006(b); Dai, 2007). Recently, the concept of rough 3-valued Lukasiewicz algebras was proposed and studied in (Dai, 2008).

At the same time, research in approximation reasoning has taken a new turn after the advent of rough set theory. There has been a variety of approaches for rough sets, including (Lin et al., 1996; Duentzsch, 1997; Liao, 2000; Sen et al., 2002; Fan et al., 2002; Dai, 2005; Dai et al., 2005). Pawlak proposed a logic based on decision system, called decision logic (Pawlak, 1991), which is a special kind of classical two-valued logic. Lin and Liu introduced two operators similar to the “possibility” and “necessity” operators in modal logic and

constructed a first order rough logic in (Lin et al., 1996). Fan, Liao and Yao modified the decision logic by the method of modal logic and fuzzy logic in (Fan et al., 2002(a); Fan et al., 2002(b)). As a matter of fact, majority of studies about logics for rough set theory have great relations with all kinds of modal logics. Readers who want to check the preceding judgment can refer to (Liao, 2000).

It is well known that the search for relationship between logic and algebra goes back to the inventions of Boolean and his follows (Rasiowz, 1974). Those investigations yielded what we now call Boolean algebra. The close links between classical logic and the theory of Boolean algebras has been known for a long time. Consequently, a question naturally arose: what are the logics that correspond to the rough algebras? Unfortunately, there are few studies about logics for rough set theory by algebraic approach. Duentzsch presents a logic corresponding to regular double Stone algebras (Duentzsch, 1997). The interconnections between the logic and regular double Stone algebras was discussed, but the logic itself including axioms, inference rules, soundness and completeness were not discussed. Banerjee and Chakraborty proposed an infinite propositional logic corresponding to pre-rough algebras in (Banerjee et al., 1996). In a subsequent paper, Banerjee has established a relationship between the finite fragment of the logic and 3-valued Lukasiewicz logic (Banerjee, 1997). They are Hilbert-type formulations with axioms and rules of inference. The implication operator in the logics is interpreted as “rough inclusion” and so the bi-implication turns out to be “rough equality”. But, no suitable implication could be obtained to develop a Hilbert-type system that would be sound and complete relative to the class of all topological quasi-Boolean algebras which are more initial structures. Sen and Chakraborty proposed sequent calculi for topological quasi-Boolean algebra and pre-rough algebras (Sen et al., 2002). We tried to study the logics with rough Stone algebraic semantics and rough double Stone algebraic semantics (Dai, 2005; Dai et al., 2005).

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