Chapter 12
Further Considerations of Classification-Oriented and Approximation-Oriented Rough Sets in Generalized Settings

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ABSTRACT

Rough sets can be interpreted in two ways: classification of objects and approximation of a set. From this point of view, classification-oriented and approximation-oriented rough sets have been proposed. In this paper, the author reconsiders those two kinds of rough sets with reviewing their definitions, properties and relations. The author describes that rough sets based on positive and negative extensive relations are mathematically equivalent but it is important to consider both because they obtained positive and negative extensive relations are not always in inverse relation in the real world. The difference in size of granules between union-based and intersection-based approximations is emphasized. Moreover, the types of decision rules associated with those rough sets are shown.

INTRODUCTION

The usefulness and efficiency of rough sets in analyses of data, information, decision and conflict have been demonstrated in the literatures (Lin, 1989a; Pawlak, 1991; Alpigini et al., 2002; Inuiguchi et al., 2003a; Wang et al., 2003). Rough set approaches have been developed mainly under equivalence relations. In order to enhance the ability of the analysis, as well as from the mathematical interests, rough sets have been generalized by many researchers (for example, (Lin, 1989a; Lin, 1989b; Dubois et al., 1990; Dubois et al., 1992; Lin, 1992; Ziarko, 1993; Yao et al., 1996;

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Bonikowski, 1998; Greco et al., 1999; Słowiński et al., 2000; Greco et al., 2003; Inuiguchi et al., 2003b; Inuiguchi et al., 2003c)). Among listed references, Ziarko (1993) generalized rough sets by parameterizing the accuracy while the others generalized rough sets by extending the equivalence relation of approximation space. In this paper, we concentrate on the latter generalizations.

Rough sets are often applied to analysis of decision tables, a collection of examples about object classifications by means of condition attributes. Considering classical rough set approaches to analysis of decision tables, the equivalence relation induced from the equality of condition attribute values implies that attributes are all nominal. Because of this assumption, unreasonable results for human intuition have been exemplified when some attributes are ordinal (Greco et al., 1999). To overcome the unreasonableness caused by the ordinal property, the dominance-based rough set approach has been proposed by Greco et al. (1999). Moreover, when the decision table includes missing values, the classical rough set approach does not work sufficiently. The generalization of rough sets is an interesting topic not only in practical point of view but also in mathematical point of view. Along this direction, rough sets have been generalized under neighborhood systems (Lin, 1989a; Lin, 1989b; Lin, 1992), similarity relations (Słowiński et al., 2000; Inuiguchi et al., 2003b), covers (Bonikowski, 1998; Inuiguchi et al., 2003b) and general relations (Yao et al., 1996) .The neighborhood systems (Lin, 1989a; Lin, 1989b; Lin, 1992) are the most general case that includes all these generalizations. Those results demonstrate a very far reaching generalization.

Considering applications of rough sets in the generalized setting, the interpretation of rough sets plays an important role. This is because any mathematical model cannot be properly applied without its interpretation. Two interpretations have been implicitly considered in the classical rough sets. One is rough set as classification of objects into positive, negative and boundary regions of a given concept. The other is rough set as approximation of a set by means of elementary sets. Those different interpretations are found in names 'positive region' (resp. 'possible region') and 'lower approximation' (resp. upper approximation') in the classical rough sets. The former rough sets are called *classification-oriented* while the latter rough sets are called *approximation-oriented*. The generalizations of rough sets under those interpretations have been proposed by Inuiguchi (Inuiguchi, 2004).

In this paper, these two kinds of rough sets are reconsidered. Reviewing the definitions and fundamental properties, we remark a connection between positively and negatively extensive relationships and the difference of the sizes of granules in union-based and intersection-based approximations. Moreover, we investigate the types of decision rules corresponding to two kinds of rough sets. We newly give the decision rules corresponding to possible and conceivable regions and upper approximations. We demonstrate the differences and similarities of classification- and approximation-oriented rough sets by a numerical example.

Classical rough sets are briefly introduced. Then define classification- and approximation-oriented rough sets and give the fundamental properties. The relationships between those two kinds of rough sets are shown. We also discuss the types of decision rules corresponding to the two kinds of rough sets. The differences of those rough sets are illustrated by a numerical example. Finally, we give the concluding remarks.

THE CLASSICAL ROUGH SETS

Let R be an equivalence relation in the finite universe U, i.e., $R \subseteq U \times U$. In rough set literature, R is referred to as an indiscernibility relation and a pair (U, R) is called an approximation space. By the equivalence relation R, U can be partitioned