

# Chapter 11

## Approximations in Rough Sets vs Granular Computing for Coverings

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### ABSTRACT

*Rough set theory is an important technique in knowledge discovery in databases. Classical rough set theory proposed by Pawlak is based on equivalence relations, but many interesting and meaningful extensions have been made based on binary relations and coverings, respectively. This paper makes a comparison between covering rough sets and rough sets based on binary relations. This paper also focuses on the authors' study of the condition under which the covering rough set can be generated by a binary relation and the binary relation based rough set can be generated by a covering.*

### INTRODUCTION

The rough set theory, proposed by Pawlak (Pawlak, 1982, 1991) in 1982, is a recent approach for reasoning about data. This theory depends basically on a certain topological structure and has achieved a great success in many fields of real life applications (Lashin, Kozae, Khadra, & Medhat, 2005; Pawlak, 1991; Polkowski & Skow-

ron, 1998). Combined with other complementary concepts such as fuzzy sets, statistics, and logical data analysis, rough sets have been exploited in hybrid approaches to improve the performance of data analysis tools (Masulli & Petrosino, 2006).

A key notion in Pawlak rough set theory is an equivalence relation. An equivalence relation, i.e., a partition, is the simplest formulation of the lower and upper approximation. However, the requirement of an equivalence relation in Pawlak rough set model seems to be a very restrictive

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condition that may limit the applications of the rough set model. Thus one of the main directions of research in rough set theory is naturally the generalization of the Pawlak rough set approximations. For example, general binary relations were used in the neighborhood systems introduced by Lin (Lin, 1989). Lin also suggested that one may use the neighborhood systems in stead of the equivalence classes. Moreover, many interesting and meaningful extensions have been made based on binary relations (Chen, Zhang, Yeung, & Tsang, 2006; Kondo, 2005, 2006; Lin, 1989, 1992; Lin, Huang, Liu, & Chen, 1990; Lin & Liu, 1994; Liu, 2006, 2008) and coverings (Bonikowski, Bryniarski, & Wybraniec, 1998; Yeung, Chen, Tsang, Lee, & Wang, 2005; Zhu, 2007; Zhu & Wang, 2006, 2007) respectively, but they did not study the relationships between these two types of rough sets. In this paper we make a comparison between covering rough sets and rough sets based on binary relations. The paper aims to study the mathematical structure of covering-based rough sets. We show that the second, third and fifth type of upper approximations can be generated by some binary relations. Under additional assumption, the first, fourth type of upper approximations and covering lower approximation can also be generated by some binary relations. Through this study, we can have a clearer vision of rough sets and a deeper insight of possible applications of rough sets. Furthermore, our study simplifies the reasoning in covering-based rough sets.

The paper is organized as follows. In Section 2 we extend the definition of rough sets to any relation and study the uniqueness of binary relations to generate rough sets. In Section 3 and Section 4 we show that covering lower and upper approximations can be generated by a reflexive and transitive relation under additional assumption. Section 5 and Section 6 make a comparison between covering upper approximations and rough sets upper approximations based on binary relations. We show that the second and third type of upper approximations can be generated by some

binary relation. In Section 7 we study the condition under which the fourth covering upper approximation coincides with the upper approximation generated by a reflexive and transitive relation. Section 8 considers the relationships between the fifth covering upper approximation and the upper approximation generated by a reflexive and transitive relation. Section 9 concludes the paper.

## ROUGH SETS BASED ON BINARY RELATIONS

Let  $U$  be a non-empty finite set of objects called the universe and  $P(U)$  be the power set of  $U$ . Recall that a binary relation  $R$  on  $U$  is referred to as reflexive if for all  $x \in U$ ,  $xRx$ ;  $R$  is referred to as symmetric if for all  $x, y \in U$ ,  $xRy$  implies  $yRx$ ;  $R$  is referred to as transitive if for all  $x, y, z \in U$ ,  $xRy$  and  $yRz$  imply  $xRz$ .  $R$  is an equivalence relation if it is reflexive, symmetric and transitive.

Several researches had pointed out the necessity to introduce a more general approach by considering an arbitrary binary relation (or even an arbitrary fuzzy relation in two universes)  $R \subseteq U \times U$  in the set  $U$  of objects instead of an equivalence relation (Bonikowski, Bryniarski, & Wybraniec, 1998; Chen, Zhang, Yeung, & Tsang, 2006; Kondo, 2005, 2006).

Suppose  $R$  is an arbitrary relation on  $U$ . With respect to  $R$ , we can define the left and right neighborhoods of an element  $x$  in  $U$  as follows:

$$l(x) = \{y \mid y \in U, yRx\} \text{ and } r(x) = \{y \mid y \in U, xRy\},$$

respectively. Clearly, if  $R$  is symmetric, then  $l(x) = r(x)$ . The left (or right) neighborhood  $l(x)$  (or  $r(x)$ ) becomes an equivalence class containing  $x$  if  $R$  is an equivalence relation. For an arbitrary relation  $R$ , by substituting equivalence class  $[x]_R$  with right neighborhood  $r(x)$ , we define the operators  $\underline{R}$  and  $\bar{R}$  (Lin, 1992) from  $P(U)$  to itself by

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