Chapter 20
Phase Unwrapping Using Energy Minimization Methods for MRI Phase Image

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ABSTRACT

Phase Unwrapping (PU) is reconstruction of absolute phase data from its wrapped phase. The absolute phase cannot be extracted from the wrapped phase data directly. Without phase noise, singularity, and aliasing problems, the phase information can be unwrapped easily. However, the phase data are always contaminated by noise and discontinuities, making the PU process more complicated. Therefore, a suitable PU algorithm is required to address the problems properly. In this method, the energy difference between neighborhood pixels in level 3 is counted, followed by getting the probability value to obtain its total fringes. The capability of the proposed method to unwrap simulated and actual MRI phase images is also demonstrated. In actual MRI phase image, PU can be implemented for water and fat separation.

INTRODUCTION

The PU algorithm can be divided into two main categories, i.e., Local and Global methods. The local method performs the unwrapping of a pixel based on its neighborhood, whereas the global method sees all the entire pixels which will be unwrapped (Ghiglia & Pritt, 1998).

Some of the local PU algorithms which have been developed are Goldstein's Branch-Cut, Quality-Guided Path Following, Mask-Cut, and Flynn's Minimum Discontinuity (Ghiglia & Pritt, 1998). Global PU method finds out the solution by changing the problem into a differential

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discret equation. In this case, the PU problems are formulated as a form of Minimum L^p-Norm. When p = 2 is used, it produces the Least-Square method. This approach minimizes the square difference between the solution gradient and the measured one which then produces a 'smooth' solution. The Least-Square method is expressed in the unweighted and weighted forms (Ghiglia & Pritt, 1998) (Zebker & Lu, 1998). Generally, the unweighted Least-Square PU focuses the unwrapping process to the inconsistent part rather than to its surrounding, which leads to an error. These limitations can be solved by using weighting factors (Pritt, 1996). Classic Multigrid method which will be introduced in the numeric perspective is followed by the description of this application method in solving the PU problems. Then, the PU progressive Multigrid is to increase the performance of the classic Multigrid (Suksmono & Hirose, 2006) (Dewi, et al., 2005). PU method using the Graph Cut has been also developed. In the Graph Cut approach, PU is conducted by counting the source and sink energy neighborhood pixels (Bioucas-Dias & Valadao, 2007).

In the previous work, we has been done implemented this methods in synthetics image and InSAR image (Adi et al., 2009). In this paper we will do unwrapping process by looking at the probability of four neighborhood pixels configuration. To verify this method we use synthetic and actual MRI image. Then the PU result is implemented for water and fat separation. The research of it has been done by using multigrid PU (Dewi et al., 2005). In the end, we will see how much is the effect of PU to the water and fat separation.

THEORY

The information about the value of 2π *phase cycle* in the absolute phase disappears. In this case, the fringe of phase process causes a sudden jump of artificial phase surrounds the edges. Mathematically, the concept of the estimation of the absolute phase image from a wrapped phase can be formulated as follows (Ghiglia & Pritt, 1998):

$$\varphi^u(i,j) = \varphi^w(i,j) + k(i,j).2\pi \tag{1}$$

where ϕ^u is the estimated absolute phase, ϕ^w is the *principal value* of the wrapped phase, and *k(i,j)* is an integer PU algorithm to be determined.

A. Energy Minimization

The statistical mechanic method uses the probabilistic information by calculating the difference energy in the four neighborhood pixels (Nishimori, 2001; Pryce & Bruce, 1995; Nicholls & Tan, 2006; Suksmono & Hirose, 2001). Four neighborhood pixels configuration to calculate the difference energy is showed in Figure 1. General formulation a four neighborhood pixel energy is defined by:

$$E = \sum_{i,j} \{(d\varphi_{ij}^w) + 2\pi.(dk_{ij})\}^2 \tag{2}$$

The initial energy (E_I) of neighborhood pixels configuration in level 3 is (Kusworo Adi, et al, 2009):

$$\begin{aligned} E_{INITIAL} = \sum_{n=1}^{3} \sum_{i,j} [\{&(\varphi_{i+n,j}^w - \varphi_{ij}^w) + (\varphi_{i-n,j}^w - \varphi_{ij}^w) + \\ &(\varphi_{i,j+n}^w - \varphi_{ij}^w) + (\varphi_{i,j-n}^w - \varphi_{ij}^w)\} + \\ &\{(k_{i+n,j} - k_{ij}) + (k_{i-n,j} - k_{ij}) + \\ &(k_{i,j+n} - k_{ij}) + (k_{i,j-n} - k_{ij})\}.2\pi]^2 \end{aligned} \tag{3}$$

So we can calculate the final energy (E_F) according to the equation:

$$\begin{aligned} E_{FINAL} = \sum_{m=1}^{3} \sum_{i,j} [\{&(\varphi_{i+m,j}^w - \varphi_{ij}^w) + (\varphi_{i-m,j}^w - \varphi_{ij}^w) + \\ &(\varphi_{i,j+m}^w - \varphi_{ij}^w) + (\varphi_{i,j-m}^w - \varphi_{ij}^w)\} + \\ &\{(k_{i+m,j} - k'_{ij}) + (k_{i-m,j} - k'_{ij}) + \\ &(k_{i,j+m} - k'_{ij}) + (k_{i,j-m} - k'_{ij})\}.2\pi]^2 \end{aligned} \tag{4}$$