

## Chapter 3.12

# Metaheuristic Search with Inequalities and Target Objectives for Mixed Binary Optimization Part I: Exploiting Proximity

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### **ABSTRACT**

*Recent adaptive memory and evolutionary metaheuristics for mixed integer programming have included proposals for introducing inequalities and target objectives to guide the search. These guidance approaches are useful in intensification and diversification strategies related to fixing subsets of variables at particular values, and in strategies that use linear programming to generate trial solutions whose variables are induced to receive integer values. In Part I (the present paper), we show how to improve such approaches by new inequalities that dominate those previously proposed and by associated target objectives that underlie the creation of both inequalities and trial solutions. Part I focuses on exploiting inequalities in target solution strategies by including partial vectors and more general target objectives. We also propose procedures for generating target objectives and solutions by exploiting proximity in original space or projected space. Part II of this study (to appear in a subsequent issue) focuses on supplementary linear programming models that exploit the new inequalities for intensification and diversification, and introduce additional inequalities from sets of elite solutions that enlarge the scope of these models. Part II indicates more advanced approaches for generating the target objective based on exploiting the mutually reinforcing notions of reaction and resistance. Our work in the concluding segment, building on the foundation laid in Part I, examines ways our framework can be exploited in generating target objectives, employing both older adaptive memory ideas of tabu search and newer ones proposed here for the first time.*

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Box 1.

$$\text{(MIP)} \begin{cases} \text{Minimize} & x_0 = fx + gy \\ \text{subject to} & (x, y) \in Z = \{(x, y) : Ax + Dy \geq b \\ & x \text{ integer} \end{cases}$$

### 1. NOTATION AND PROBLEM FORMULATION

We represent the mixed integer programming problem in the form shown in Box 1.

We assume that  $Ax + Dy \geq b$  includes the inequalities  $U_j \geq x_j \geq 0, j \in N = \{1, \dots, N\}$ , where some components of  $U_j$  may be infinite. The linear programming relaxation of (MIP) that results by dropping the integer requirement on  $x$  is denoted by (LP). We further assume  $Ax + Dy \geq b$  includes an objective function constraint  $x_0 \leq U_0$ , where the bound  $U_0$  is manipulated as part of a search strategy for solving (MIP), subject to maintaining  $U_0 < x_0^*$ , where  $x_0^*$  is the  $x_0$  value for the currently best known solution  $x^*$  to (MIP).

The current paper focuses on the zero-one version of (MIP) denoted by (MIP:0-1), in which  $U_j = 1$  for all  $j \in N$ . We refer to the LP relaxation of (MIP:0-1) likewise as (LP), since the identity of (LP) will be clear from the context.

In the following we make reference to two types of search strategies: those that fix subsets of variables to particular values within approaches for exploiting strongly determined and consistent variables, and those that make use of solution targeting procedures. As developed here, the latter solve a linear programming problem  $LP(x', c')$ <sup>1</sup> that includes the constraints of (LP) (and additional bounding constraints in the general (MIP) case) while replacing the objective function  $x_0$  by a linear function  $v_0 = c'x$ . The vector  $x'$  is called a *target solution*, and the vector  $c'$  consists of integer coefficients  $c'_j$  that seek to induce assignments  $x_j = x'_j$  for different variables with varying degrees of emphasis.

We adopt the convention that each instance of  $LP(x', c')$  implicitly includes the (LP) objective of minimizing the function  $x_0 = fx + gy$  as a secondary objective, dominated by the objective of minimizing  $v_0 = c'x$ , so that the true objective function consists of minimizing  $\omega_0 = Mv_0 + x_0$ , where  $M$  is a large positive number. As an alternative to working with  $\omega_0$  in the form specified, it can be advantageous to solve  $LP(x', c')$  in two stages. The first stage minimizes  $v_0 = c'x$  to yield an optimal solution  $x = x''$  (with objective function value  $v_0'' = c'x''$ ), and the second stage enforces  $v_0 = v_0''$  to solve the residual problem of minimizing  $x_0 = fx + gy$ .<sup>2</sup>

A second convention involves an interpretation of the problem constraints. Selected instances of inequalities generated by approaches of the following sections will be understood to be included among the constraints  $Ax + Dy \geq b$  of (LP). In our definition of  $LP(x', c')$  and other linear programs related to (LP), we take the liberty of representing the currently updated form of the constraints  $Ax + Dy \geq b$  by the compact representation  $x \in X = \{x : (x, y) \in Z\}$ , recognizing that this involves a slight distortion in view of the fact that we implicitly minimize a function of  $y$  as well as  $x$  in these linear programs.<sup>3</sup>

To launch our investigation of the problem (MIP:0-1) we first review previous ideas for generating guiding inequalities for this problem in Section 2 and associated target objective strategies using partial vectors and more general target objectives in Section 3. We then present new inequalities in Section 4 that improve on those previously proposed. The fundamental issue of creating the target objectives that can be used to

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