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Chapter XI

2D Image Matrix-Based Discriminator

ABSTRACT

This chapter presents two straightforward image projection techniques — two-dimensional (2D) image matrix-based principal component analysis (IMPCA, 2DPCA) and 2D image matrix-based Fisher linear discriminant analysis (IMLDA, 2DLDA). After a brief introduction, we first introduce IMPCA. Then IMLDA technology is given. As a result, we summarize some useful conclusions.

INTRODUCTION

The conventional PCA and Fisher LDA are both based on vectors. That is to say, if we use them to deal with the image recognition problem, the first step is to transform original image matrices into same dimensional vectors, and then rely on these vectors to evaluate the covariance matrix and determine the projector. Two typical examples, the famous eigenfaces (Turk & Pentland, 1991a, 1991b) and fisherfaces (Swets & Weng, 1996; Belhumeur, Hespanha, & Kriegman, 1997) both follow this strategy. The drawback of this strategy is obvious. For instance, considering an image of 100×100 resolution, its corresponding vector is 10,000-dimensional. To perform K-L transform or Fisher linear discriminant on basis of such high-dimensional image vectors is a time-consuming process. What's more, the high dimensionality usually leads to singularity of the within-class covariance matrix, which causes trouble for calculation of optimal discriminant vectors (projection axes).

In this chapter, we will develop two straightforward image projection techniques; that is, 2DPCA and *2DLDA*, to overcome the weakness of the conventional PCA and LDA as applied in image recognition. Our main idea is to directly construct three image covariance matrices, including *image between-class*, *image within-class* and *image total scatter matrices*; and then, based on them, perform PCA or Fisher LDA. Since the scale of image covariance matrices is same as that of images and the within-class image covariance matrix is usual nonsingular, thus, the difficulty resulting from high dimensionality and singular case are artfully avoided. We will outspread our idea in the following sections.

2D IMAGE MATRIX-BASED PCA

IMPCA Method

Differing from PCA and KPCA, IMPCA, which is also called 2DPCA (Yang & Yang, 2002; Yang, Zhang, Frangi, & Yang, 2004), is based on 2D matrices rather than 1D vectors. This means that we do not need to transform an image matrix into a vector in advance. Instead, we can construct an *image covariance matrix* directly using the original image matrices, and then use it as a generative matrix to perform principal component analysis.

The *image covariance* (*scatter*) *matrix* of 2DPCA is defined by:

$$\mathbf{G}_t = E(\mathbf{A} - E\mathbf{A})^T (\mathbf{A} - E\mathbf{A}) \tag{11.1}$$

where $\mathbf{A}$ is an $m \times n$ random matrix representing a generic image. Each training image is viewed as a sample generated from the random matrix $\mathbf{A}$. It is easy to verify that $\mathbf{G}_t$ is an $n \times n$ non-negative definite matrix by its construction.

We can evaluate $\mathbf{G}_t$ directly using the training image samples. Suppose that there are a total of M training image samples, the j_{th} training image is denoted by an $m \times n$ matrix $\mathbf{A}_j$ ($j = 1, 2, \ldots, M$), and the mean image of all training samples is denoted by $\bar{\mathbf{A}}$. Then, $\mathbf{G}_t$ can be evaluated by:

$$\mathbf{G}_t = \frac{1}{M} \sum_{j=1}^{M} (\mathbf{A}_j - \bar{\mathbf{A}})^T (\mathbf{A}_j - \bar{\mathbf{A}}) \tag{11.2}$$

The projection axes of 2DPCA, $\mathbf{X}_1, \ldots, \mathbf{X}_d$, are required to maximize the total scatter criterion $J(\mathbf{X}) = \mathbf{X}^T \mathbf{G}_t \mathbf{X}$ and satisfy the orthogonal constraints; that is:

$$\begin{cases} \{\mathbf{X}_1, \cdots, \mathbf{X}_d\} = \arg\max J(\mathbf{X}) \\ \mathbf{X}_i^T \mathbf{X}_j = 0, \ i \neq j, \ i, j = 1, \ \ldots, \ d \end{cases} \tag{11.3}$$

Actually, the optimal projection axes, $\mathbf{X}_1, \ldots, \mathbf{X}_d$, can be chosen as the orthonormal eigenvectors of $\mathbf{G}_t$ corresponding to the first d largest eigenvalues.

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