



## Chapter X

# Complete Kernel Fisher Discriminant Analysis

### ABSTRACT

*This chapter introduces a complete kernel Fisher discriminant analysis (KFD) that is a useful statistical technique applied to biometric application. We first describe theoretical perspective of KPCA. Then, a new KFD algorithm framework, KPCA plus LDA, is given. Afterwards, we discuss the complete KFD algorithm. Finally, the experimental results and chapter summary are given.*

### INTRODUCTION

Over the last few years, kernel-based learning machines — that is, SVMs, KPCA, and kernel Fisher discriminant analysis (KFD) — have aroused considerable interest in the fields of pattern recognition and machine learning (Müller, Mika, Rätsch, Tsuda, & Schölkopf, 2001). KPCA was originally developed by Schölkopf (Schölkopf, Smola, & Müller, 1998) while KFD was first proposed by Mika (Mika, Rätsch, Weston, Schölkopf, & Müller, 1999). Subsequent research saw the development of a series of KFD algorithms (Baudat & Anouar, 2000; Roth & Steinhage, 2000; Mika, Rätsch, & Weston, 2003; Yang, 2002; Lu, Plataniotis, & Venetsanopoulos, 2003; Xu, Zhang, & Li, 2001; Billings & Lee, 2002; Cawley & Talbot, 2003). The KFD algorithms developed by Mika, Billings, and Cawley (Mika, Rätsch, & Weston, 2003; Billings & Lee, 2002; Cawley & Talbot, 2003) are formulated for two classes, while those of Baudat, Roth, and Yang (Baudat & Anouar, 2000; Roth & Steinhage, 2000; Yang, 2002) are formulated for multiple classes. Because of its ability to extract the most discriminatory non-linear features, KFD has been found to be very effective in many real-world applications.

KFD, however, always encounters the *ill-posed* problem in its real-world applications (Mika, Rätsch, & Weston, 2003; Tikhonov & Arsenin, 1991). A number of regularization techniques that might alleviate this problem have been suggested. Mika (Mika, Rätsch, & Weston, 2003; Mika, Rätsch, & Weston, 1999) used the technique of making the inner product matrix non-singular by adding a scalar matrix. Baudat (Baudat & Anouar, 2000) employed the QR decomposition technique to avoid the singularity by removing the zero eigenvalues. Yang (2002) exploited the PCA plus LDA technique adopted in fisherface (Belhumeur, Hespanha, & Kriegman, 1997) to deal with the problem. Unfortunately, all of these methods discard the discriminant information contained in the null space of the within-class covariance matrix, yet this discriminant information is very effective for the SSS problem (Liu & Yang, 1992; Chen, Liao, & Ko, 2000; Yu & Yang, 2001; Yang & Yang, 2001; Yang & Yang, 2003). Lu (Lu, Plataniotis, & Venetsanopoulos, 2003) has taken this issue into account and presented kernel direct discriminant analysis (KDDA) by generalization of DLDA (Yu & Yang, 2001).

In real-world applications, particularly in image recognition, there are a lot of SSS problems where the number of training samples is less than the dimension of feature vectors. For kernel-based methods, due to the implicit high-dimensional nonlinear mapping determined by kernel, almost all problems are turned into SSS problems in *feature space* (actually, all problems will become SSS problems as long as the dimension of nonlinear mapping is large enough). Actually, KPCA and KFD are inherently in tune with the linear feature extraction techniques like PCA and Fisher LDA for SSS problems. Eigenface (Turk & Pentland, 1991) and fisherface (Belhumeur, Hespanha, & Kriegman, 1997) typically are PCA and LDA techniques for SSS problems. They are essentially carried out in the space spanned by all  $M$  training samples by virtue of the SVD technique. Like eigenface and fisherface, KPCA and KFD are also performed in all training samples' spanned space. This inherent similarity makes it possible to improve KFD using the state-of-the-art LDA techniques.

LDA has been well studied and widely applied to SSS problems in recent years. Many LDA algorithms have been proposed. The most famous method is fisherface, which is based on a two-phase framework of PCA plus LDA. The effectiveness of this framework in image recognition has been broadly demonstrated. Recently, the theoretical foundation for this framework has been laid (Yang & Yang, 2003). Besides, many researchers have dedicated to search for more effective discriminant subspaces. A significant result is the finding that there exists crucial discriminative information in the null space of the within-class scatter matrix (Liu & Yang, 1992; Chen, Liao, & Ko, 2000; Yu & Yang, 2001; Yang & Yang, 2001, 2003). In this chapter, we call this kind of discriminative information *irregular* discriminant information, in contrast with *regular* discriminant information outside of the null space.

KFD would be likely to benefit in two ways from the state-of-the-art LDA techniques. One is the adoption of a more concise algorithm framework, and the other is that it would allow the use of *irregular* discriminant information. This chapter seeks to improve KFD in these ways: first of all, by developing a new KFD framework, KPCA plus LDA, based on a rigorous theoretical derivation in Hilbert space. Then, a complete KFD algorithm (CKFD) is proposed based on the framework. Unlike current KLD algorithms, CKFD can take advantage of two kinds of discriminant information: *regular and irregular*. Finally, CKFD was used in face recognition and handwritten numeral recognition. The experimental results are encouraging.

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