

Chapter 13

My Equations are the Same as Yours!

Computer Aided Assessment Using a Gröbner Basis Approach

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ABSTRACT

In this chapter we explain how computer aided assessment (CAA) can automatically assess an answer that consists of a system of equations. In particular, we will use a computer algebra system (CAS) and Buchberger's Algorithm to establish when two systems of equations are the "same."

INTRODUCTION

Our primary concern in this chapter is the assessment of mathematics with computers, and in particular the ability of CAA software to provide automatic feedback in a formative setting to support students. We focus on mathematical methods that allow for the assessment and provision of feedback on questions involving systems of equations. In Section 1 we consider word exercises.

These, we argue, are an important component in mathematics education. In Section 2 we discuss computer aided assessment of mathematics and the role of feedback in CAA. Section 3 covers the mathematics underpinning systems of equations. We discuss what it means to solve equations, and when two systems are the same. We examine Euclid's algorithm and Gaussian elimination and then move onto Gröbner basis techniques for manipulating systems of polynomial equations. Section 4 combines the previous topics by explaining how we can automatically assess the modelling

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component of answers to word exercises using the STACK computer aided assessment system.

Word Exercises and Modelling

The motivation for the work discussed in this chapter is the desire to improve the ability of computer aided assessment to deal with questions that involve modelling. There is a general consensus among researchers of mathematics education that problem solving and modelling are important skills for students of the mathematical sciences. Blum and Niss [1991] offered an overview of arguments for and against such training, their view being summed up in the following quote:

Mastering mathematics can no longer be considered equivalent to knowing a set of mathematical facts. It requires also the mastering of mathematical processes, of which problem solving - in the broadest sense - occupies a predominant position.

In *Mathematical Discovery* [Pólya, 1981, pg. 59], Polya asserts that word problems in particular deserve a special place on the mathematical curriculum:

I hope that I shall shock a few people in asserting that the most important single task of mathematical instruction in the secondary schools is to teach the setting up of equations to solve word problems. Yet there is a strong argument in favor of this opinion. In solving a word problem by setting up equations, the student translates a real situation into mathematical terms; he has an opportunity to experience that mathematical concepts may be related to realities, but such relations must be carefully worked out.

In their report *Mathematics for the European Engineer - A Curriculum for the Twenty-First Century* [Mustoe & Lawson, 2002], SEFI, the Société Européenne pour la Formation des Ingénieurs, wrote

The ability to formulate a mathematical model of a given physical situation, to solve the model, interpret the solution and refine the model is a key aspect of the mathematical development of an engineer.

Word exercises serve as an introduction to modelling and problem solving in general as they require a student to transcribe a situation described in words to a system of equations [Sangwin, 2010]. Such transcription is not as straightforward as one may assume; in [1981] Clement et al. gave the example of the Students-and-Professors Problem:

Example 1

Write an equation for the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.

When this was given to 150 calculus level students, 37% answered incorrectly with $6S=P$ accounting for two thirds of all errors, [Clement et al, 1981].

Given that students can experience such difficulties with problems that contain very little in the way of technical mathematics, a CAA system that has the ability to pose and mark word exercises could be a very useful tool to a teacher. Asking students to transcribe a given situation into a system of equations creates a number of challenges when it comes to the system marking responses. Choices of variables, co-ordinate systems or origin may lead to situations where correct answers differ greatly in appearance. It is important that a CAA system can handle deftly such differences; marking a correct answer as incorrect could have serious repercussions for a student.

While the student and professors problem contains an element of *modelling*, there are other types of word problems. A comprehensive classification of 1097 high-school algebra story problems was developed by [Mayer, 1981]. The following is a typical example.

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