Chapter 4.12
Application of Uncertain Variables to Knowledge-Based Resource Distribution

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ABSTRACT

The chapter concerns a class of systems composed of operations performed with the use of resources allocated to them. In such operation systems, each operation is characterized by its execution time depending on the amount of a resource allocated to the operation. The decision problem consists in distributing a limited amount of a resource among operations in an optimal way, that is, in finding an optimal resource allocation. Classical mathematical models of operation systems are widely used in computer supported projects or production management, allowing optimal decision making in deterministic, well-investigated environments. In the knowledge-based approach considered in this chapter, the execution time of each operation is described in a nondeterministic way, by an inequality containing an unknown parameter, and all the unknown parameters are assumed to be values of uncertain variables characterized by experts. Mathematical models comprising such two-level uncertainty are useful in designing knowledge-based decision support systems for uncertain environments. The purpose of this chapter is to present a review of problems and algorithms developed in recent years, and to show new results, possible extensions and challenges, thus providing a description of a state-of-the-art in the field of resource distribution based on the uncertain variables.

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INTRODUCTION

Among many theories of uncertainty (Klir, 2006) developed for different applications the uncertain variables introduced by Bubnicki (2001a, 2001b) may be considered as a useful tool for modeling expert's knowledge in knowledge-based decision systems. In the definition of the uncertain variable $\bar{x}$ we consider two soft properties: "$\bar{x} \cong x$" which means "$\bar{x}$ is approximately equal to x" or "x is the approximate value of $\bar{x}$," and "$\bar{x} \tilde{\in} D_x$" which means "$\bar{x}$ approximately belongs to the set D_x" or "the approximate value of $\bar{x}$ belongs to D_x." The *uncertain variable* $\bar{x}$ is defined by a set of values X (real number vector space), the function $h(x) = v(\bar{x} \cong x)$ (i.e., the *certainty index* that $\bar{x} \cong x$, given by an expert) and the following definitions for $D_x, D_1, D_2 \subseteq X$:

$$v(\bar{x} \tilde{\in} D_x) = \max_{x \in D_x} h(x)$$

$$v(\bar{x} \tilde{\notin} D_x) = 1 - v(\bar{x} \tilde{\in} D_x),$$

$$v(\bar{x} \tilde{\in} D_1 \vee \bar{x} \tilde{\in} D_2) \\ = \max\{v(\bar{x} \tilde{\in} D_1), v(\bar{x} \tilde{\in} D_2)\},$$

$$v(\bar{x} \tilde{\in} D_1 \wedge \bar{x} \tilde{\in} D_2) \\ = \begin{cases} \min\{v(\bar{x} \tilde{\in} D_1),\ v(\bar{x} \tilde{\in} D_2)\} & \text{for} \quad D_1 \cap D_2 \neq \varnothing \\ 0 & \text{for} \quad D_1 \cap D_2 = \varnothing. \end{cases}$$

The function $h(x)$ is called a *certainty distribution*. Let us consider a plant with the input vector $u \in U$ and the output vector $y \in Y$, described by a relation $R(u,y;x) \subset U \times Y$ (*relational knowledge representation*) where the vector of unknown parameters $x \in X$ is assumed to be a value of an uncertain variable described by the certainty distribution $h(x)$ given by an expert. If the relation R is not a function, then the value u determines a set of possible outputs $D_y(u;x) = \{y \in Y: (u,y) \in R(u,y;x)\}$. For the requirement $y \in D_y \subset Y$ given by a user, we can formulate the following decision problem: For the given $R(u,y;x)$, $h(x)$ and D_y one should find the decision u^* maximizing the certainty index that the set of possible outputs approximately belongs to D_y (i.e., belongs to D_y for an approximate value of $\bar{x}$). Then

$$u^* = \arg\max_{u \in U} v[D_y(u;\bar{x}) \tilde{\subseteq} D_y] \\ = \arg \max_{u \in U} \max_{x \in D_x(u)} h(x)$$

where $D_x(u) = \{x \in X: D_y(u;x) \subseteq D_y\}$. It is easy to see that u^* maximizes $v[u \tilde{\in} D_u(\bar{x})]$ where $D_u(x)$ is a set of all u such that the implication $u \in D_u(x) \rightarrow y \in D_y$ is satisfied. The uncertain variables are dedicated to analysis and decision problems (Bubnicki, 2002, 2004a) in a class of systems containing a decision plant described by a relational knowledge representation with unknown parameter characterized by an expert.

An important example for such a class of decision plants may be a *complex of operations*. It consists of operations characterized by their execution times, and the execution time of a particular operation depends on the amount of a resource allocated to the operation. All the operations use the same kind of a resource which is continuous and may be distributed among operations in any way. In the knowledge-based approach under consideration, this relationship has a form of a relation and an unknown parameter in this relation is assumed to be a value of an uncertain variable characterized by an expert. The decision problem consists then in finding a resource allocation to the operations optimizing a given *performance index* and satisfying the user's requirement typically concerning the execution time of the whole set of operations. Because the resource distribution is based on uncertain knowledge, certainty indexes should be used in decision problem formulations.

Complexes of operations with operations characterized by their execution times are decision plants different than *activity networks* widely used in production or project management (e.g., Banaszak & Jozefowska, 2003). In these networks, the set of activities (production operations or project tasks) is depicted by a graph and the activities are represented by arcs assigned prob-

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