

# Stiffness Modeling and Analysis of Passive Four-Bar Parallelogram in Fully Compliant Parallel Positioning Stage

*X. Jia, Tianjin University, China*

*Y. Tian, Tianjin University, China*

*D. Zhang, Tianjin University, China*

*J. Liu, Hebei University of Technology, China*

---

## ABSTRACT

*In order to investigate the influence of the stiffness of the compliant prismatic pair, a planar four-bar parallelogram, in a fully compliant parallel mechanism, the stiffness model of the passive compliant prismatic pair in a compliant parallel positioning stage is established using the compliant matrix method and matrix transformation. The influences of the constraints and the compliance of the connecting rods on the flexibility characteristics of the prismatic pair are studied based on the developed model. The relative geometric parameters are changed to show the rules of the stiffness variation and to obtain the demands for simplification in the stiffness modeling of the prismatic pair. Furthermore, the finite element analysis has been conducted to validate the analytical model.*

*Keywords:* Compliant Mechanism, Finite Element Analysis, Matrix Transformation, Stiffness Matrix, Stiffness Modeling

---

## INTRODUCTION

As a combination of parallel manipulator and compliant mechanism, compliant parallel mechanisms can provide the merits of both mechanisms, such as high stiffness, wide response band, vacuum compatibility, clean room compatibility, zero error accumulation,

no backlash, friction free and no need for lubrication. Consequently, compliant parallel mechanisms have a potential in various fields where an ultra-precision manipulation system is first and foremost required (Ryu & Gweon, 1997a; Ryu & Gweon, 1997b; Yi, Na, & Chung, 2002; Tian, Shirinzadeh, & Zhang, 2008; Tian, Shirinzadeh, & Zhang, 2009).

The stiffness characteristic of the flexure-based parallel mechanism plays an important

DOI: 10.4018/ijimr.2011010104

role in the practical applications. It will affect the workspace, load-carrying capacity, dynamic behavior and the positioning accuracy of the entire mechanism. A spatial compliant parallel mechanism with high stiffness can provide a more dexterous and precise motion and possess high natural frequency. Currently, the research on the stiffness of flexure-based mechanism mainly focuses on the modeling and analysis of whole mechanism or individual flexure hinges including elliptical flexure hinges, corner-filletted hinges, and right circular hinges, etc. Lobontiu and Garcia (2003) provided an analytical method for stiffness calculations of planar compliant mechanism with single-axis flexure hinges based on the strain energy and Castigliano's displacement theorem. By discussing about the relationship between the input/output stiffness and mechanism geometric parameters, a stiffness optimization procedure was proposed. Koseki, Tanikawa, and Koyachi (2000) and Yu, Bi, and Zong (2002) obtained the stiffness model based on the matrix method and the equilibriums of displacements and forces. Pham and Chen (2005) analyzed the stiffness of a six-DOF flexure parallel mechanism based on the simplified modeling method. Xu and Li (2006) introduced a more straightforward approach, which employed only one kind of transformation matrix, to derive the stiffness matrix of an orthogonal compliant parallel micromanipulator. Dong, Sun, and Du (2008) established the stiffness equation of a 6-DOF high-precision parallel mechanism via assembling stiffness matrices and formulating constraint equations. By simplifying the flexure hinge as an ideal revolution joint with a linear torsional spring and the equation equilibriums of forces, Tian, Shirinzadeh, & Zhang (2008) derived the stiffness model of a compliant five-bar mechanism. Based on the model, the influences of the position on the stiffness and the position of the end-effector point are discussed.

Stiffness modeling of individual flexure hinge is the premise to analyze the compliance of spatial flexure-based mechanism. In general,

the single axis flexure hinge is utilized as the compliant revolute joint and the flexure-based planar four-bar parallelogram as compliance prismatic pair. Paros and Weisbord (1965), Smith, Badami, and Dale (1997), and Tian, Shirinzadeh, and Zhang (2010) have done a number of works on the stiffness modeling of flexure hinge with different notches, and established the analytic formulations. However, when the stiffness of compliant prismatic pair was involved in the above literatures, the simplifications have commonly been adopted. The flexure hinges and the right/left links in the parallelogram were regarded as elastic bodies and the compliance of the connecting rods on the upper/lower terminal of the parallelogram was neglected. In addition, the research was primarily focused on the stiffness along the translation direction of compliant prismatic pair, although Yang, Yin, and Ma (2005) formulated the stiffness equation of compliant prismatic pair using the energy method.

In order to solve above mentioned problems, the mechanical structure of a 3-DOF flexure-based parallel positioning stage is described briefly in this paper. The stiffness of the passive compliant prismatic pair is modeled analytically and the influences of the geometric dimensions, restriction condition and the compliance of the connecting rod on the stiffness are investigated and discussed. Finally, the proposed analytical model is validated by FEM simulation.

## MECHANICAL DESIGN

A modified Delta parallel mechanism which consists of only revolute joints, firstly presented by Tsai and Stamper (1996), is chosen as the prototype of compliant mechanism. The revolute joints only offer rotation about one axis that is similar to the single-axis flexure hinge. Therefore, replacing the revolute joints in the modified Delta mechanism with single-axis right circular flexure hinges and adjusting the

16 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: [www.igi-global.com/article/stiffness-modeling-analysis-passive-four/52059](http://www.igi-global.com/article/stiffness-modeling-analysis-passive-four/52059)

## Related Content

---

### Robust Adaptive Unscented Particle Filter

Li Xue, Shesheng Gao and Yongmin Zhong (2013). *International Journal of Intelligent Mechatronics and Robotics* (pp. 55-66).

[www.irma-international.org/article/robust-adaptive-unscented-particle-filter/90300](http://www.irma-international.org/article/robust-adaptive-unscented-particle-filter/90300)

### Cognitive Computation: An Exact Bayesian Inference Stochastic Machine

Marvin Faix, Emmanuel Mazer, Raphaël Laurent, Mohamad Othman Abdallah, Ronan Le Hy and Jorge Lobo (2020). *Robotic Systems: Concepts, Methodologies, Tools, and Applications* (pp. 906-929).

[www.irma-international.org/chapter/cognitive-computation/244042](http://www.irma-international.org/chapter/cognitive-computation/244042)

### A Scene-Based Episodic Memory System for a Simulated Autonomous Creature

Elisa C. Castro and Ricardo R. Gudwin (2013). *International Journal of Synthetic Emotions* (pp. 32-64).

[www.irma-international.org/article/scene-based-episodic-memory-system/77655](http://www.irma-international.org/article/scene-based-episodic-memory-system/77655)

### Adjustment of Medical Observations Influenced by Emotional State

Firas Zekri, Afef Samet Ellouze and Rafik Bouaziz (2018). *International Journal of Synthetic Emotions* (pp. 1-22).

[www.irma-international.org/article/adjustment-of-medical-observations-influenced-by-emotional-state/209422](http://www.irma-international.org/article/adjustment-of-medical-observations-influenced-by-emotional-state/209422)

### Opinion Mining of Twitter Events using Supervised Learning

Nida Hakak and Mahira Kirmani (2018). *International Journal of Synthetic Emotions* (pp. 23-36).

[www.irma-international.org/article/opinion-mining-of-twitter-events-using-supervised-learning/214874](http://www.irma-international.org/article/opinion-mining-of-twitter-events-using-supervised-learning/214874)