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Appendix A

On the Fuzzy Tools

Abstract

The first part of this appendix presents three approaches in defining the fuzzy version (generalization) of the mathematical graph structure: graphs with fuzzy vertices, graphs with fuzzy edges, and graphs with fuzzy vertices and edges. Their advantages and shortcomings are discussed briefly. Fuzzy graphs are observed in the light of fuzzy relations theory, and as a generalization of the notion of random graph. In the second part, we generalize some fuzzy algebraic structures towards not only [0, 1] valued, but lattice, poset, and relational structured valued structures. It is exciting to see how powerful a modeling tool they are, and also to see how classical results continue to hold as but a special case of the new results.

Graph Structures

In this section we overview some basics of graph theory and introduce the pivotal results that supported the idea of investigating fuzzy graph structures.

The term graph is used in two paradigmatically different ways in the contemporary mathematical theory. The first meaning has its roots back in the time of Descartes and denotes depicting of functional dependencies. The work in the

fuzzification of this particular notion, as well as in its applications, has been more than significant in the last 5 years (Berthold & Huber, 1998; Zadeh 1971, 1994). The applications show that fuzzy graphs are suitable for dealing with modeling of dependencies when uncertainty of different types, which is useful in cases where the human factor and reasoning cannot be neglected.

The huge number of references dealing with fuzzy functional graphs may cause misunderstandings and may give a faulty impression that fuzzy graph structures (in the sense of fuzzy versions of the crisp mathematical graph structures) have been studied in depth.

The discussion on the present status of research in fuzzy graphs can be carried out on two different levels. If we take into account the research done in the field of fuzzy relations and random graphs (special cases of fuzzy graphs), then it can be stated that it has attained broad scientific interest. On the other hand, there is scarce literature dealing with approaches towards the unified notion of fuzzy graphs that are basically developed in order to serve as frameworks for describing certain work in the applicable and application domain, especially in the engineering areas, in artificial intelligence, and cognitive sciences in general.

Most of the literature dates from beginning of the 1990s. There are no particular indices that fuzzy graphs have been of significant interest to the theoretical mathematical society. What we will try to do in this appendix is to summarize the *theoretical* attempts in the domain and give ideas for further work on the concept of fuzzy graph, which we find extremely appealing.

In this appendix the theory will be presented via general, oriented graphs. The same theory can easily be rewritten to fit the nonoriented graphs, and whenever appropriate, comments will be made to cover those graphs as well.

We start our consideration of fuzzy graphs by stating the traditional, crisp definition of the term graph.

Definition A.1. Let V be a given set and let:

$$E \subseteq V \times V.$$

Then the pair (relational structure):

$$G = G(V,E),$$

is called graph, where V is the set of objects (vertices), that is, the carrier of the structure, and E is its set of edges, that is, the adjacency relation.