Appendix C: Important Mathematical Definitions

Convolution. The convolution $f(x, y) = g(x, y) * h(x, y)$ of two 2-dimensional functions $g(x, y)$ and $h(x, y)$ is defined as:

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(u,v)h(x-u, y-v)dudv$$

Correlation. The cross-correlation function $c(x, y) = g(x, y) \star h(x, y)$ of two 2-dimensional functions $g(x, y)$ and $h(x, y)$ is defined as:

DOI: 10.4018/978-1-61520-785-5.ch014

$$c(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g^*(u,v)h(x+u,y+v)dudv$$

Corresponding relations. Relation of correlation *c(x, y)* and convolution *:

$c(x,\ y) = g^*(x,\ y)\ ^*h(-x,-y).$

If *F* denotes Fourier transform, and *G* and *H* denote Fourier-transformations of *g* and *h*, then:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\ g(x,y)\ |^2\ dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\ G(k_x,k_y)\ |^2\ dk_x dk_y$$ (Plancherel or Rayleigh theorem),

$F[g(x,y)\ \bigstar\ g(x,y,)]=|\ G(k_x,k_y)\ |^2$ (Wiener-Khinchin theorem),
$F[g(x,y)\ ^*h(x,y,)]=G(k_x,k_y)H(k_x,k_y)$ (See also MacLennan, 1990.)