Chapter 12
A Stochastic Perturbation Algorithm for Inventory Optimization in Supply Chains

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ABSTRACT

In recent years, simulation optimization has attracted a great deal of attention because simulation can model the real systems in fidelity and capture complex dynamics. Among numerous simulation optimization algorithms, Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm is an attractive approach because of its simplicity and efficiency. Although SPSA has been applied in several problems, it does not converge for some. This research proposes Augmented Simultaneous Perturbation Stochastic Approximation (ASPSA) algorithm in which SPSA is augmented to include presearch, ordinal optimization, non-uniform gain, and line search. Performances of ASPSA are tested on complex discrete supply chain inventory optimization problems. The tests results show that ASPSA not only achieves speed up, but also improves solution quality and converges faster than SPSA. Experiments also show that ASPSA is comparable to Genetic Algorithms in solution quality (6% to 15% worse) but is much more efficient computationally (over 12x faster).

INTRODUCTION

The emergence of simulation optimization over the last decade has made optimization of large, complex, systems easier by providing an ability to obtain good performance of the system without requiring complete knowledge of the systems or exact analysis of the objective function. Unlike analytical techniques, simulation can accommodate arbitrary stochastic elements, and generally allow modeling of all of the complexities and dynamics of real world systems without unrealistic simplifying assumptions. In recent years, commercial simulation software also has integrated optimization packages (see, e.g., Glover, 1996; Carson and Maria, 1997; Fu, 2002; April et al., 2003 for detail discussion). However, many commercial software solutions commonly adopt

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naive genetic algorithms (GAs) and simulated annealing (SA), which may be time consuming and inefficient for large optimization problems. Hence, there is a need for more research for specific optimization problems using simulation (Carson and Maria, 1997; Fu 2002).

Simultaneous Perturbation Stochastic Approximation (SPSA) (Spall, 1992, 1999) is a gradient based simulation optimization method. Consider θ_k, the decision vector at iteration k. The essence of SPSA is to use the gradient $g_k(\theta_k)$ to compute the decision vector at the next iteration as follows: $\theta_{k+1} = \theta_k - a_k\, g_k\,(\theta_k)$.

The method relies on a computationally efficient estimate of the gradient and calculates gradient by perturbing in all directions simultaneously, as shows in Equation(1).

$$g_k(\theta) = \begin{bmatrix} \dfrac{y(\theta_k + c_k\Delta_k) - y(\theta_k - c_k\Delta_k)}{2c_k\Delta_{k1}} \\ \vdots \\ \dfrac{y(\theta_k + c_k\Delta_k) - y(\theta_k - c_k\Delta_k)}{2c_k\Delta_{kp}} \end{bmatrix} \tag{1}$$

where $\Delta_k = [\Delta_{k1}, \Delta_{k2},..., \Delta_{kp}]^T$ represents a perturbation vector. SPSA has been used in various fields; for example, see Fu and Hill (1997), Hopkins (1997), Bhatnagar et al. (2001) for more details. Although SPSA was designed for continuous optimization problems, Gerencsėr and Hill (1999) also found out that SPSA algorithms can also be applied on discrete optimization problems. However, Wan et al. (2005) found SPSA can be noisy and biased for continuous supply chain inventory optimization problems, which are vital for enterprises holding significant inventories, which can cost anywhere between 20 to 40 percent of their value. In addition, most of research in discrete supply chain inventory optimization problems still predominantly use simulated annealing (SA) and genetic algorithms (GAs) (see Daniel et al., 2005a, 2005b), which tend to require inordinate amount of computational time for large supply network optimization. There is a need for developing efficient algorithms for supply chain inventory optimization.

Therefore, in this paper, Augmented Simultaneous Perturbation Stochastic Approximation (ASPSA) is proposed based on SPSA, and its application for discrete supply chain inventory optimization problems is studied. ASPSA augments five techniques to improve the performance of SPSA, and they are: 1) Ordinal Optimization (OO), 2) Presearch, 3) Line search, 4) Memory of best solution, and 5) Non-uniform gain.

The remainder of this paper is organized as follows. Section 2 describes ASPSA algorithm. In Section 3, we present experimental design and Section 4 details the numerical results of the experiments. Finally we present our conclusion in Section 5.

AUGMENTED SIMULATANEOUS PERTURBATION STOCHASTIC APPROXIMATION (ASPSA) ALGORITHM

The list of notations we will use in the paper is as following:

i**:** Index of decision variable
θ: Decision vector
p**:** Dimension of decision vector
k**:** Iteration index
$\boldsymbol{y_k}$**:** Noisy objective
$\boldsymbol{\Delta_k}$**:** Perturbation vector
n**:** Full simulation run length
$\boldsymbol{n_s}$**:** Shorter simulation run length ($n_s < n$)
m**:** Pre-search number
l**:** Line search number

It should be emphasized that in simulation, we can only get noisy objective y_k.

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