

Chapter 11

Synchronization of Oscillators

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ABSTRACT

A simple approach is proposed in this chapter to get started on the synchronization of oscillators study. The basics are given in the beginning such that the reader can get quickly familiar with the main concepts which lead to many kinds of synchronization configurations. Chaotic synchronization is next addressed and is followed by the stability of the synchronization issue. Finally, a short introduction of the influence of noise on the synchronization process is mentioned.

INTRODUCTION

Nonlinearity in physics has become a wide topic under investigation in many areas of scientific research such as biology, engineering, social sciences and others. It is a challenge for many researchers to address many issues related to nonlinearity which induces somehow regular and chaotic behaviors in systems governed by non linear differential or partial differential equations. In what follows an attempt is made to give some basics in synchronization of oscillators. Most of the issues have been addressed in a simple and standard manner. Both regular and chaotic dynamical systems have been taken into account such that any beginner in this topic will find the way to derive fundamental quantities and parameters that characterize the system they are dealing with.

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FUNDAMENTALS OF THE SYNCHROIZATION

Kuramoto's Model

One of the most important basic phenomena in sciences is synchronization which was discovered by C. Huyghens (Pecora & Carroll, 1990; Pikovsky, Rosenblum, & Kurths, 2001; Rulkov et al., 1995; Kuramoto, 1984). Synchronization is a phenomenon which means there is an adjustment of the frequencies of periodic self-sustained oscillators through a weak interaction. The weakness of the interaction is to some extent a requirement. The frequencies adjustment is also referred to as phase locking or frequency entrainment.

Many types of synchronizations can be distinguished depending on the dynamic property of the system under investigation. One might have identical synchronization, generalized synchronization and phase synchronization for chaotic and non-chaotic systems (i.e. regular and chaotic systems) Synchronization might also happen in a system of two or many identical or not chaotic systems. It also may be seen as the appearance of a relation relating the phases of interacting systems or between the phase of a system and that of an external force. The synchronization phenomenon is not to be confused with resonance phenomenon and synchronous variation of two variables which does not lead to synchronization. In many areas of science collective synchronization phenomena have been observed. These areas overlaps biology, chemistry, physical and social systems as examples. This phenomenon is mainly observed when two or many oscillators lock on to a common frequency despite differences in the frequency of the individual oscillators. A popular model known as Kuramoto model has been developed and can be found in many textbooks and articles in the literature. Many methods such as the stability analysis can be used to address the synchronization phenomenon. The Kuramoto model consists of N oscillators whose dynamics are governed by the following equations:

$$\dot{\theta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) \quad (1)$$

where θ_i is the phase of the i th oscillator, ω_i is its natural frequency, K is positive and stands for the coupling gain.

The oscillators are said to synchronize if

$$\dot{\theta}_j - \dot{\theta}_i \rightarrow 0 \text{ as } t \rightarrow \infty \quad \forall i, j = 1, \dots, N \quad (2)$$

The oscillators can also be said to synchronize when $\theta_j - \theta_i$ becomes constant asymptotically as the time goes on. If we represent the oscillators on a circle by points that move with the same angular frequency, then the phase difference which means the angular distance between these points remain constant over the time. Thus, one can define the order parameter r that measures the phase coherence of the oscillators population and takes values between 0 and 1 inclusively as follow

$$re^{i\psi} = \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} \quad (3)$$

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