Chapter 6
Synchronization of Chaotic Oscillators

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ABSTRACT

In this chapter, the author reviews the variety of forms that the synchronization of the dynamics of mutually coupled or unidirectionally driven chaotic oscillators can display. The aim is to provide a presentation of a background of knowledge on fundamental physics and mathematics that the author expects to be useful in the development of the application of chaos synchronization to telecommunications and cryptography.

INTRODUCTION

One of the most remarkable properties of nonlinear systems is its ability to display chaotic dynamics. The fundamental feature of a system displaying this kind of dynamics is its sensitivity to initial conditions. This means that the dynamics of such system is unpredictable, and consequently irregular and aperiodic. Such features may appear, in principle, undesirable and something to be avoided, at least in engineering applications. However, research on the dynamics of coupled and driven chaotic oscillators, among other issues, has proven these features of nonlinear dynamics to be interesting and potentially useful.

When two chaotic oscillators are mutually coupled, or when one of such oscillators drives another, they may display the phenomenon of chaos synchronization (Pikovsky et al., 2001; González-Miranda, 2004). The study of the synchronization of chaotic oscillators has become a topic on its own within the field of nonlinear dynamics and chaos because of both, its scientific interest as a phenomenon characteristic of nonlinear systems, and its potential applications in many fields. Among them, we have the secure transmission of information in telecommunications, which is based on the use of the unstable and irregular behavior of chaotic systems to conceal the information being transmitted.

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A relevant feature of chaos synchronization is the variety of forms that it can display. In some of them the motion of the synchronized oscillations is strongly correlated, in others this correlation is quite faint, while there are types of synchronization that are somewhere in the middle, and presenting different qualitative features.

Some forms of synchronization may be observed, in principle, on any kind of chaotic systems. Ordered from the strongest to the faintest, being the first and the second equally strong, these are: Identical Synchronization (Fujisaka & Yamada,1983; Pecora & Carroll, 1990), Generalized Synchronization (Rulkov et al., 1995), lag synchronization (Rosenblum et al., 1997), Phase Synchronization (Rosenblum et al., 1996), and Amplitude Envelope Synchronization (González-Miranda, 2002a). A given system is able to display several of these forms of synchronization, is a series of stages of chaotic synchronization, starting from the faintest, as the strength of the coupling is progressively increased.

Moreover, there are other types of synchronization, which are special in the sense they are linked to some particular property, or feature of the chaotic oscillator involved. These include Anticipated Synchronization (Voss, 2000), Marginal Synchronization (González-Miranda, 1996a), and Multistable Synchronization (González-Miranda, 1996b).

The mission of this chapter is to review these different forms of synchronization to provide a systematic presentation of a material that is fundamental for the application of chaos theory to cryptography for secure communications. Each of these forms of synchronization of chaos will be described in qualitative and quantitative form, techniques needed for its observation and measure will be given, as well as conditions for their occurrence.

CHAOTIC OSCILLATORS AND SYNCHRONIZATION LAYOUTS

Deterministic chaos is ubiquitous in nature and has been observed in many fields of physics, chemistry, biology, geology and astrophysics. In particular, it has been observed in systems of interest for telecommunications such as electric circuits and lasers.

An example of electric circuit is the one introduced by Matsumoto et al. (1985) and its many variants (Chua et al., 1993; Bilotta et al., 2007; Gomes & King, 1992). Other interesting chaotic electric circuits exist, some of them reproducing chaotic systems borrowed from other fields. For example the celebrated Lorenz (1963) and Rössler (1976) systems, have been implemented as electric circuits by Cuomo and Oppenheim (1993b) and by Carroll (1995), respectively.

Practically, all kinds of lasers can be prepared to display chaotic behavior (Harrison & Biswas, 1986). In particular, chaotic dynamics has been observed in experiments performed on such a variety of lasers as: CO_2 lasers (Midavaine et al., 1985), semiconductor lasers (Mukai & Otsuka, 1985), Nd:Yttrium Aluminum Garnet lasers (Bracikowski & Roy, 1991) and erbium-doped fiber ring lasers (VanWiggeren & Roy, 1998).

These systems are usually modeled by means of sets of ordinary differential equations. For example, the electric circuit by Matsumoto et al. (1985) which contains a nonlinear resistor, a resistor of resistance R, two capacitors of capacitances C_1 and C_2, and an inductance L, can be described by a system of three non-linear equations on the variables x, y and z, describing the voltages of the capacitors 1 and 2 and the current in the inductance respectively. This reads