Chapter 1
Introduction

In this chapter, we provide brief introductions to the basic concepts of global optimization, evolutionary computation, and swarm intelligence. The necessity of solving optimization problems is outlined and various problem types are reported. A rough classification of established optimization algorithms is provided, followed by the historical development of evolutionary computation. The three fundamental evolutionary approaches are briefly presented, along with their basic features and operations. Finally, the reader is introduced to the field of swarm intelligence, and a strong theoretical result is concisely reported to justify the necessity for further development of global optimization algorithms.

WHAT IS OPTIMIZATION?

Optimization is a scientific discipline that deals with the detection of optimal solutions for a problem, among alternatives. The optimality of solutions is based on one or several criteria that are usually problem- and user-dependent. For example, a structural engineering problem can admit solutions that primarily adhere to fundamental engineering specifications, as well as to the aesthetic and operational expectations of the designer. Constraints can be posed by the user or the problem itself, thereby reducing the number of prospective solutions. If a solution fulfills all constraints, it is called a *feasible solution*. Among all feasible solutions, the *global optimization* problem concerns the detection of the optimal one. However, this is not always possible or necessary. Indeed, there are cases where suboptimal solutions are acceptable, depending on their quality compared to the optimal one. This is usually described as *local optimization*, although the same term has been also used to describe local search in a strict vicinity of the search space.

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A modeling phase always precedes the optimization procedure. In this phase, the actual problem is modeled mathematically, taking into account all the underlying constraints. The building blocks of candidate solutions are translated into numerical variables, and solutions are represented as numerical vectors. Moreover, a proper mathematical function is built, such that its global minimizers, i.e., points where its minimum value is attained, correspond to optimal solutions of the original problem. This function is called the *objective function*, and the detection of its global minimizer(s) is the core subject of global optimization. Instead of minimization, an optimization problem can be equivalently defined as maximization by inverting the sign of the objective function. Without loss of generality, we consider only minimization cases in the book at hand.

The objective function is accompanied by a domain, i.e., a set of feasible candidate solutions. The domain is delimited by problem constraints, which need to be quantified properly and described mathematically using equality and inequality relations. In the simplest cases, constraints are limited to bounding boxes of the variables. In harder problems, complex relations among the variables must hold in the final solution, rendering the minimization procedure rather complicated.

Analytical derivation of solutions is possible for some problems. Indeed, if the objective function is at least twice continuously differentiable and has a relatively simple form, then its minimizers are attained by determining the zeros of its gradient and verifying that its Hessian matrix is positive definite at these points. Apparently, this is not possible for functions of high complexity and dimensionality or functions that do not fulfill the required mathematical assumptions. In the latter case, the use of algorithms that approximate the actual solution is inevitable. Such algorithms work iteratively, producing a sequence of search points that has at least one subsequence converging to the actual minimizer.

Optimization has been an active research field for several decades. The scientific and technological blossoming of the late years has offered a plethora of difficult optimization problems that triggered the development of more efficient algorithms. Real-world optimization suffers from the following problems (Spall, 2003):

a. Difficulties in distinguishing global from local optimal solutions.
b. Presence of noise in solution evaluation.
c. The "curse of dimensionality", i.e., exponential growth of the search space with the problem's dimension.
d. Difficulties associated with the problem's constraints.

The different nature and mathematical characteristics of optimization problems necessitated the specialization of algorithms to specific problem categories that share common properties, such as nonlinearity, convexity, differentiability, continuity, function evaluation accuracy etc. Moreover, the inherent characteristics of each algorithm may render it more suitable either for local or global optimization problems. Such characteristics include, among others, stochasticity, potential for parallelization in modern computer systems and limited computational requirements.

Today, there is a rich assortment of established algorithms for most problem types. Nevertheless, even different instances of the same problem may have different computational requirements, leaving space for development of new algorithms and the improvement of established ones. Consequently, there will be an ongoing need for new and more sophisticated ideas in optimization theory and applications.

In the next section, we put the optimization problem into a mathematical framework, which allows the distinction between different problem types, and we identify major categories of optimization algorithms, related to the topics of the book at hand.