Chapter 15
Insights and Conclusions

ABSTRACT

This chapter concludes the combinatorics analysis, highlighting its evolution, modern applications, and prospects. The discipline finds applications in many fields, demonstrating its interdisciplinary value. The importance of multiple representations in teaching combinatorics is emphasized to promote deeper understanding and critical thinking. New research areas such as asymptotic combinatorics, combinatorial bioinformatics, and complex network theory are also explored, showcasing their expanding relevance. Advanced examples are presented, including Stirling's approximation for factorials, percolation theory for studying diffusion phenomena, and using Feynman diagrams in quantum physics and statistical mechanics. Furthermore, the chapter examines the application of combinatorics in social network models, workforce shift optimization, and combinatorial game theory. The chapter raises questions about the discipline's future, suggesting that combinatorics represents a proper "way of thinking" capable of tackling complex problems across various fields.

INSIGHTS AND CONCLUSIONS

Those who dare to teach must never stop learning
John Cotton Dana

After providing a general overview of the discipline and its objectives, we will analyse several techniques and strategies for solving combinatorial problems. The previous examples have allowed us to see the practical application of the theories and methods studied. The sub-disciplines and their examples gave us a detailed look at the various specialized areas within combinatorics, demonstrating the breadth and diversity of this field. We explored how combinatorial methods are crucial in various fields, including algorithms (Mehlhorn & Sanders, 2007) and graph theory, as well as recursive counting and ordinary generating functions. Combinatorial games and random combinations have demonstrated the importance of combinatorics in playful and casual contexts.

The didactics of combinatorics offered insights on effectively teaching this discipline, emphasising the importance of a clear and structured approach to facilitate learning.

Additional guidance is provided by Maher and Martino (1996a). They believe it is essential that students are encouraged to construct and use more representations while working on problems. Using different tools to build and express ideas enables students to create connections between various representations, thereby enhancing their understanding of the math they are learning. This allows observers (teachers and classmates) to understand better the ideas formulated by students.

At this point, a question arises: What are the prospects? We know that combinatorics continues to evolve, with new applications emerging in various fields. Numerous open problems in combinatorics pose fascinating challenges for researchers. These include fundamental issues in more traditional topics, such as combinatorial counting and graph theory (Van Steen, 2010; Bollobás, 2011), but also in fields yet to be explored, such as asymptotic combinatorics, a branch of combinatorics that deals with the study of the behaviour of sequences of numbers or functions, particularly combinatorial counts, when the parameters in question tend towards infinity or algorithmic combinatorics that relies on artificial intelligence (Russell & Norvig, 2016) and Machine Learning. In addition, new research directions are emerging

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in areas such as topological combinatorics, complex network theory, and combinatorial bioinformatics. The most classic example of asymptotic combinatorics refers to the Stirling formula:

$$n! \approx \sqrt{2\pi n} \cdot (\frac{n}{e})^n$$

It provides an approximation for the factorial of a large number. The following program displays the comparison of exact factorials and Stirling approximations on a logarithmic scale for *n* up to 50 and prints the error.

Program 150 Stirling

```
    import math
    import matplotlib.pyplot as plt
    def stirling_approximation(n):
    return math.sqrt(2 * math.pi * n) * (n / math.e) ** n
    def factorial(n):
    return math.factorial(n)
    def compare_factorials_graph(max_n):
    n_values = list(range(1, max_n + 1))
    exact_factorials = [factorial(n) for n in n_values]
    stirling_approximations = [stirling_approximation(n) for n in n_values]
    relative_errors = [abs(exact_factorials[i] - stirling_approximations[i]) / exact_factori-
als[i] * 100 for i in range(max_n)]
    # Plot comparing exact factorials and Stirling approximation
    plt.figure(figsize=(14, 8))
    plt.plot(n_values, exact_factorials, label=”Exact Factorial”, marker='o', linestyle='-',
color='b')
    plt.plot(n_values, stirling_approximations, label=”Stirling Approximation”, marker='x',
linestyle='--', color='r')
    plt.yscale('log')
    plt.xlabel('n')
    plt.ylabel('Value')
    plt.title('Comparison between Exact Factorial and Stirling Approximation')
    plt.legend()
    plt.grid(True)
    plt.show()
    # Plot of the relative error
    plt.figure(figsize=(14, 8))
    plt.plot(n_values, relative_errors, label=”Relative Error (%)”, marker='s', linestyle='-',
color='g')
    plt.yscale('log')
    plt.xlabel('n')
    plt.ylabel('Relative Error (%)')
    plt.title('Relative Error of Stirling Approximation compared to Exact Factorials')
    plt.legend()
    plt.grid(True)
    plt.show()
    # Modify the value of max_n to see the comparison up to a specific value of n
    compare_factorials_graph(50)
```

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