Chapter 13
Methodological Choices

ABSTRACT

The chapter explores combinatorics teaching through a structured methodological approach, considering theoretical and practical aspects. Jean Piaget's idea that combinatorial thinking emerges during the formal operational stage (ages 11-12) is discussed, although later studies have challenged this theory. Vergnaud's distinction between conceptualization and symbolization highlights the importance of an interpretative model based on operational invariants. Lockwood's model identifies three essential components of combinatorics: mathematical formulas, counting processes, and sets of outcomes. It emphasizes the need for a seamless transition between these elements for a deeper understanding. The suggested teaching approach includes progressiveness, diversification, and cooperation, stressing the use of visual and digital tools such as diagrams and interactive software to facilitate learning. The chapter demonstrates how combinatorics can be applied to real-world contexts through practical examples, such as password problems, sports tournaments.

METHODOLOGICAL CHOICES

What matters is not the amount of things you do, but the efficiency of each individual action

Tim Ferriss

Didactic experimentation in combinatorics requires a careful and structured methodological approach. Many mathematicians, psychologists, and pedagogues have contributed (Biggs, 2002; Anderson, 2012; Tucker, 2012; Zazkis & Liljedahl, 2019; Mazur, 2022; Sandefur et al., 2022), and it is not easy to move forward with the formulations and suggestions, sometimes being opposed. In his studies, the well-known developmental psychologist Jean Piaget theorized that combinatorial thinking could not be formed before a stage of growth indicated by the scholar as the formal operative stage (11-12 years), an approach later criticized based on further studies. Several studies that are not specific to combinatorics can be easily included.

Consider the following statement by Vergnaud, father of conceptual field theory: "The distinction between conceptualizing and symbolizing is essential, up to the point where the understanding of words and sentences by different persons, particularly students and teachers, is not simply a binary relationship signifier/signified, but a ternary one with the interpretation privileged by operational invariants". The distinction between conceptualizing and symbolizing is fundamental. Students and teachers' understanding of combinatorial problems and solutions is not reduced to a simple binary relationship between the signifier (e.g., a combinatorial formula) and the meaning (the underlying mathematical concept). Rather, it is a ternary relationship that also includes interpretation based on operational invariants, i.e., the conceptual structures individuals use to deal with combinatorial situations (Vergnaud, 1992). For example, when considering the formula for combinations, we have: the signifier, which is the formula, the meaning that is enclosed in the concept of selecting r elements from a set of n without repetition and without considering the order, the operational invariants represented by the mental strategies (and operations) that students use to interpret and apply the concept. This ternary perspective is particularly relevant in combinatorics, where students often struggle to connect symbolic formulas to underlying concepts and concrete situations.

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Another interesting model of students' combinatorial thinking states that combinatorial reasoning (English, 2005) is based on three distinct but related components: formulas, counting processes, and sets of outcomes. In this respect, we can cite the model developed by Elise Lockwood, a professor of mathematics education. She is a researcher known for her work on students' learning of combinatorics and mathematical reasoning. Her studies focus on how people understand and solve combinatorial problems, emphasising counting processes and using visual and computational representations (Lockwood, 2013; Lockwood, 2014)).

Figure 1. Lockwood model

The model focuses on three interrelated aspects of combinatorial thinking: the mathematical formulas and notations used to represent combinatorial problems, the procedures and methods employed to systematically count possibilities, and the set of all possible outcomes of a given combinatorial scenario.

The key idea of the model is that these three aspects are closely linked and that students should be able to switch between them to develop a deep understanding of combinatorics. The well-known formula permits calculating combinations of n elements taken k at a time. However, it is not enough to memorize this formula. Students need to understand its meaning and derivation and see if there is a different point of view. An example clarifies this last aspect: a password is made up of ten capital letters. How many of these 10-letter passwords contain at least 3 P? The problem is complex, and we must consider many cases. Another effective approach is to subtract those that contain 0, 1, or 2 P from all passwords. This approach could encourage students to explain the connections between formulas, processes, and outcomes.

Field research has identified that the model followed by the students follows this process:

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