Chapter 9
Combinatorics Games

ABSTRACT

This chapter examines combinatorial games, analyzing how decisions and strategies are based on the combination of different options. Combinatorial games involve creating and manipulating combinations of cards, tokens, or pieces on a board. These games focus on the organization and arrangement of available objects. Combinatorial games have a long history and are significant from both a research and educational perspective, introducing fundamental concepts such as combinations, permutations, and arrangements in a practical and accessible context. For example, in poker, players combine their cards to create the best possible hand, using combinatorics to determine probabilities. In chess, the Eight Queens Problem is a famous example of a combinatorial problem that requires placing eight queens on an 8x8 board so no queen can attack another. Other combinatorial games include Sudoku, Tower of Hanoi, and Rubik's Cube.

INTRODUCTION

Play is a free, circumscribed, uncertain, unproductive, regulated and fictive activity

Roger Caillois

Combinatorial games involve creating and manipulating combinations of elements, such as cards, pawns, or pieces on a chessboard. It is a class of abstract games involving strategies, decisions, and situations in which players must choose based on a finite set of options or configurations (Berlekamp et al., 2004; Demaine & Hearn, 2008; Hefetz et al., 2014; AA.VV., 2018; García Jiménez, 2018). These games focus on the combination and arrangement of available objects, resources, or moves, rather than the probability or randomness aspects found in many other games. These games have been played for centuries worldwide and hold considerable importance from both a research and educational perspective.

Combinatorial games introduce fundamental concepts, such as combinations, permutations, and arrangements, in a practical and accessible context. Consider card games; for example, poker requires players to combine their cards to form the best possible hand. The case history is vast, but it would be unfair to limit the analysis to "games" only; in reality, there are many applications in concrete situations. From financial investment strategies to the design of communication networks, combinatorial games play a crucial role in optimising decisions in complex situations (Niv & Patkin, 2020).

Listing all the possible uses would take too long; a few hints are enough to understand the extent of the phenomenon. In communication systems and computer networks, combinatorial games can be used to optimize data routing across a network. In the transportation and logistics industries, they can be used to optimize vehicle routes, reduce costs, and improve transportation efficiency. In statistics and experimental design, they are used to design experiments efficiently, selecting optimal combinations of variables to test. In computational biology, combinatorial games can be used to model interactions between molecules and proteins, contributing to the understanding of biological processes and the design of new drugs. In machine learning systems, combinatorial games model collaborative learning situations.

One of the main categorizations of combination games is the number of players involved. There are numerous works on games for two or more players, although many games in real life are essentially single player.

Let us examine some of the most popular combination games, beginning with those that involve more than one player and progressing to "automatic" games.

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Astragals: The talus is a small bone in the foot joint. In some animal species, such as goats and rams, this bone has particularly regular proportions, and since ancient times, it has been used as a four-sided die. These ossicles were widely spread throughout the Mesopotamian region, while in Egypt, they spread around 1300 BC. The astragals were thrown, and based on the face facing upwards, it was possible to "read" a score: 1 (Monas), 3 (Trias), 4 (Tetras), 6 (Hexas). It is worth noting that the opposite sides added up to 7, just like traditional cubic dice.

Figure 1. Astragals

A name was assigned to each side and to each of the combinations that could be obtained by throwing four astragals. Since each talus can land on one of the four faces and there are four taluses, we can calculate the total number of possible combinations with the formula $4^4 = 256$. By throwing four astragals, there are several combinations of scores. The Romans assigned names and symbolic meanings to certain notable combinations:

1. Aphrodite's Strike: 1, 3, 4, 6; The most esteemed throw, representing harmony and beauty, was associated with the goddess of love.
2. Dog Strike: 1, 1, 1, 1; Considered the worst possible throw, symbolizing misfortune or disgrace.
3. Triple Threes: 3, 3, 3, 3; A rare and significant uniform roll.
4. Tetras: 4, 4, 4, 4; A uniform roll of fours, potentially lucky or neutral depending on context.
5. Six of Venus: 6, 6, 6, 6; A high and auspicious roll, sometimes associated with Venus or good fortune.

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