Chapter 6
The Sub-Disciplines

ABSTRACT

This chapter dives into the rich diversity of combinatorics, emphasizing how it goes far beyond simple combinatorial calculation. Six important subdisciplines are presented: combinatorial design theory, focused on the construction of structures with particular properties (such as block drawings); code theory, which uses combinatorics to design and analyse codes for reliable data transmission, with examples like Hamming and Golay codes; geometric combinatorics, which investigates the combinatorial properties of geometric objects; algebraic combinatorics, which links combinatorics to algebra to solve problems through algebraic tools and includes algorithms like Jarvis for convex hulls; topological combinatorics, which creates a bridge between combinatorics and the topological properties of objects and examines discrete structures using topological methods, such as Sperner's theorem and Delaunay triangulation; and finally, experimental design, which explores the link between combinatorics and the structuring of experiments to obtain meaningful results.

INTRODUCTION

Combinatorics is the mathematics of choices

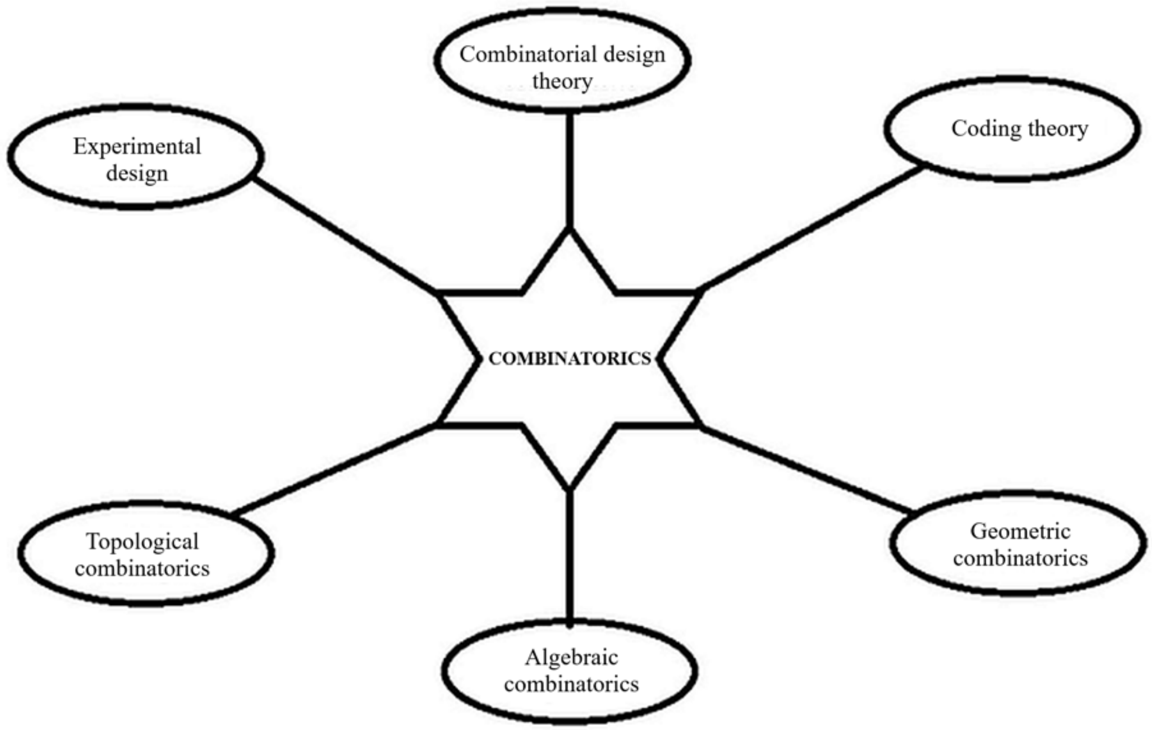
David Gale

The discipline is vast and diverse and is not limited to the better-known enumerative combinatorics (Bóna, 2015), which is closely related to combinatorics, computational combinatorics, or combinatorial game theory. It encompasses numerous subfields and has practical applications in various scientific and technological fields, including the design of web search algorithms and DNA analysis.

The sub-disciplines are numerous; those covered are represented in the figure:

DOI: 10.4018/979-8-3373-3089-1.ch006

Figure 1. Map of some sub-disciplines



Six branches extend from the central node, each representing a specific area of application or study within combinatorics:

1. *Combinatorial design theory* focuses on constructing combinatorial structures with specific properties, such as block designs.
2. *Coding theory* applies combinatorics to the design and analysis of codes for reliable information transmission.
3. *Geometric combinatorics* studies the combinatorial properties of geometric objects.
4. *Algebraic combinatorics* bridges the gap between combinatorics and algebra, utilising algebraic tools to solve complex combinatorial problems.
5. *Topological combinatorics* explores the relationship between combinatorics and the topological properties of the studied objects.
6. *Experimental design* highlights the connection between combinatorics and the planning of experiments, where multiple factors are combined to obtain meaningful and statistically valid results.

Let us examine these six sub-disciplines.

Combinatorial design theory: involves constructing statistical experiments, known as "experimental designs," that satisfy specific properties (Dukes et al., 2008; Colbourn & Mathon, 2011). Combinatorial designs allow efficient experiments that systematically evaluate the effect of different variables with a minimum number of observations. This is useful in statistics and probability theory but also in the field of experimental psychology.

In experimental psychology, combinatorial designs serve to arrange experimental setups that test multiple independent variables.