

Chapter 2

Methods

ABSTRACT

The chapter explores the fundamental concepts of combinatorics, presenting essential methods and models for the analysis of combinatorial structures. The chapter discusses classical combinatorics, focusing on permutations, combinations, and partitions. It introduces the perspective of J.G. Dubois, who emphasizes combinatorial structures rather than mere enumeration, classifying problems into selections, distributions, and partitions. The fundamental principle of product and sum, the principle of equality and inclusion-exclusion, which allow us to manage counting problems and set structures, are examined in detail. Additional combinatorial tools discussed include generating functions, which transform combinatorial problems into algebraic problems, and binomial coefficients, which are fundamental in the calculus of combinations and related to Pascal's triangle. The chapter also highlights combinatorial models such as shafts and the pigeonhole principle. Finally, the computational aspects of combinatorics are explored through algorithms.

INTRODUCTION

Combinatorics is the mathematics of patterns

Persi Diaconis

If combinatorics were a toolbox, what tools would be indispensable? It is not easy to answer because this field of mathematics is wide and varied. Given the breadth of the subject, it is not easy to exhaustively define the essential methods of combinatorics, and not all scholars propose the same approach (Jukna, 2001; Bogart, 2004). Classical combinatorics (Dubois, 1984; Roberts & Tesman, 2009; Stanley, 2011), generally focuses on problems such as Permutations (ordered arrangements of objects), Combinations (choices of subsets of objects), and Partitions (division of a set into subsets).

The purpose of combinatorics is oversimplified in these terms. Combinatorics transcends the role of a simple counting tool to become a methodological cornerstone in the decision-making process. This discipline can be effectively employed for three primary purposes, each corresponding to a distinct stage of the decision-making process.

- *Identifying Possibilities:* Firstly, combinatorics is essential for identifying and defining the number of possibilities within a given choice or probability scenario. This initial phase is critical, as an accurate understanding of the entire “solution space” is a prerequisite for an informed decision. Through tools like permutations, arrangements, and combinations, it is possible to precisely quantify the total number of possible outcomes or configurations. For instance, when calculating the probability of an event, combinatorics provides the crucial denominator—the number of all possible cases—allowing for the mapping of the universe of available options before any evaluation.
- *Narrowing Down Choices:* Subsequently, combinatorics helps to reduce this number of possibilities to facilitate a choice. While combinatorics does not inherently eliminate options, its ability to enumerate and characterize different configurations is instrumental in defining manageable subsets upon which to apply selection criteria. Knowing the exact number of alternatives that meet specific constraints or requirements allows for the analysis to be focused on a more restricted and pertinent set of options. For example, in logistics, once all possible routes

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for a delivery are calculated, combinatorics enables the identification and isolation of only those that fall within predefined time or cost parameters, making the choice easier and more targeted.

- *Verifying and Confirming Decisions*: Finally, combinatorics supports the stage of verifying and confirming the choice as the best one. After making a decision, it is crucial to assess its robustness and optimality. Combinatorial techniques can be employed to analyse alternative scenarios or to calculate the probability that the chosen decision will indeed be the most advantageous given the conditions. This allows not only for post-decision validation but also for continuous optimisation, providing a quantitative framework for comparing the chosen option with other potential configurations and for testing its resilience to variations or uncertainties.

An interesting position is that of J.G. Dubois; in his 1984 work, he explores combinatorial structures and tries to classify configurations through a particular model, shifting the focus from simple enumeration to a deeper structural analysis. Dubois suggests that we think of problems not just as “counts” but as real “structures.” Let us imagine people are organising a party. The challenge is not merely counting guests; it quickly extends to determining the optimal arrangement of tables, seating everyone, and even the sequence in which courses are served.

According to Dubois, simple combinatorial configurations can be classified into three models: *selections*, which emphasize the concept of sampling; *distributions*, related to the concept of mapping, and *partitions* or divisions of a set into subsets. This subdivision does not exactly correspond to the classical classification of combinatorics, but it offers a particular point of view that emphasizes specific aspects. The selections focus on the concept of sampling, which is related to classical combinatorics, particularly combinations. The distribution model can be associated with partitions of objects or distributions of objects across categories. Partitions are about dividing a set into disjoint subsets. This concept is certainly similar to that of classical combinatorics. However, the fundamental difference lies in the fact that Dubois emphasizes structuring the set, where each subset is treated as a distinct “class”.

In the latter case, an example can make this approach more explicit. Suppose we divide students in a school into different groupings (e.g., math, science, art, literature), where each student can belong to more than one subset. The interest lies not only in the number of ways to divide students but also in the shape of the network to which they belong and the interactions between the different groupings.

The Dubois approach offers significant advantages when dealing with complex combinatorial problems, where it is necessary to count and understand the structure and relationships between objects. It is advantageous in abstract, theoretical and interdisciplinary contexts. However, its conceptual complexity and greater computational difficulty make it less suitable for simpler situations.

In addition to Dubois' combinatorial method, other significant and non-traditional positions in combinatorics methods deserve attention. These positions differ in approaches and applications, expanding the field of combinatorics beyond classical methods (Bender & Williamson, 2013). Extreme combinatorics deals with situations in which it is necessary to determine whether a particular arrangement of objects is possible, often addressing complex problems such as those related to the configurations of graphs or sets (Katona, 1975; Erdős & Simonovits, 1982; Simonovits, 1984; Bollobás, 1998).

Enumerative combinatorics focuses on counting objects, employing techniques to determine asymptotic values.

Probabilistic combinatorics uses probability techniques to study the asymptotic properties of combinatorial elements.

Traditional methods of combinatorics focus primarily on simple problems and explicit counts.

However, extreme and enumerative combinatorics extend in more complex directions, utilising asymptotic counting and determining upper and lower bounds to understand combinatorial structures better and optimise their classification.

The topic could be further expanded and, later, several ideas will be taken up, for now, we can limit ourselves to a more traditional overview of the methods of combinatorics, but useful as it is sufficiently articulated:

1. *The counting principle* – This principle is the basis for calculating the number of ways in which objects can be organized or selected. It is based on fundamental concepts such as the principle of multiplication (Lockwood et al., 2017), the principle of addition, and the principle of inclusion-exclusion, all of which are typical examples of enumerative methods.
2. *Generating functions* – Generating functions are powerful mathematical tools that enable the manipulation of combinatorial sequences by converting complex counting tasks into algebraic expressions, thus simplifying calculations.

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