

Chapter 12

Optimal Boundary Control of String Vibrations With Multipoint Intermediate Conditions at Minimizing the Boundary Energy Integral

Vanya Barseghyan

*Institute of Mechanics, The National Academy of Sciences of the RA, Yerevan State University,
Armenia*

Svetlana Solodusha

 <https://orcid.org/0000-0001-6162-7542>

Melentiev Energy Systems Institute of Siberian Branch of the Russian Academy of Sciences, Russia

ABSTRACT

For the equation of string vibration with given initial and final conditions, we consider the problems of optimal boundary control with given multipoint conditions at different intermediate time points for the values of deflection functions and velocities of string points. The control is carried out both by displacement at one end with the other end fixed, and by displacement at the two ends of the string. The minimizable functionals are the boundary energy integrals given over the entire time interval. We propose a constructive approach to designing solutions to the considered problems of optimal boundary control of string vibrations. The construction scheme is as follows: the problems are reduced to control problems with distributed impacts and zero boundary conditions, then, using the method of separation of variables, the obtained problems are reduced to optimal control problems for ordinary differential equations with specified initial, final and multipoint intermediate conditions with appropriate quality criteria.

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INTRODUCTION

This chapter addresses the tasks of optimal boundary control of string vibrations with initial, final conditions, and specified multipoint conditions at various intermediate moments in time concerning the values of the deflection function and the velocities of points on the string. The problems of optimal boundary control are considered in two cases: when the control is implemented as a displacement at one end while the other end is fixed, and when there are displacements at both ends of the string. The functionals to be minimized are integrals of the squares of the derivatives of the boundary displacements, i.e., the integral of boundary energy, defined over the entire time interval. The goal of this work is to develop a constructive approach to constructing the function of optimal boundary control actions for managing the vibrations of the string with given initial, final conditions, and intermediate values of deflections and velocities of points on the string at various intermediate moments in time.

BACKGROUND

Mathematical models of many physical and technological processes, which have significant theoretical and practical importance, are described by the one-dimensional wave equation. In particular, the wave equation describes the vibrations of a string, links of a manipulator, level-luffing crane, aircraft wings, and many other processes. It is important to note that controlled vibrating systems are widely distributed in various theoretical and applied fields of science. Therefore, there is a need to investigate control and optimal control problems for vibrational processes described by the wave equation with various formulations, both concentrated (boundary) and distributed effects. Such problems are relevant, and many researchers pay attention to their study, particularly as discussed in works (Abdukarimov, 2013); (Abdukarimov, 2014); (Butkovsky, 1965); (Butkovsky, 1975); (Dreglea and Sidorov, 2018); (Gibkina et al., 2016); (Il'in and Moiseev, 2005); (Krabs, 1995); (Li, 2008); (Moiseev and Kholomeeva, 2013); (Sadybekov and Yessirkegenov, 2016); (Sirazetdinov, 1977); (Yang et al., 2004); (Znamenskaya, 2004). An introductory overview of the theory of controlling distributed vibrating systems is provided, particularly in works (Butkovsky, 1965); (Butkovsky, 1975); (Krabs, 1995); (Sirazetdinov, 1977); (Znamenskaya, 2004). Works (Il'in and Moiseev, 2005) and (Moiseev and Kholomeeva, 2013) (and other works by these authors and their students) are dedicated to the problem of boundary control (optimal control) of wave processes in the class of generalized solutions. In investigating these boundary control problems, the D'Alembert method is used and D'Alembert-type formulas are derived. Works (Abdukarimov, 2013) and (Abdukarimov, 2014) examine optimal boundary control tasks involving displacements at the ends of a string based on minimizing the integral of boundary energy, which transitions the vibration process of the string from an arbitrarily given initial state to a specified final state over an arbitrary sufficiently long time interval. In studies related to control and optimal control of vibrational processes, particularly, in the works cited above, various methods for solving control and optimal control problems, such as the D'Alembert method, the Fourier method, the harmonic method and the method of moments, have been used.

Many dynamic control processes with both concentrated and distributed parameters lead to the necessity of investigating multipoint boundary control and optimal control problems, where, alongside classical boundary (initial and final) conditions, multipoint intermediate (both separated and non-separated) conditions are also specified (Ashchepkov, 1981); (Barseghyan, 2012); (Barseghyan, 2016a); (Barseghyan, 2016b); (Barseghyan, 2018); (Barseghyan, 2019); (Barseghyan, 2020); (Barseghyan,

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