Chapter 4
Nature Inspired Methods for Multi-Objective Optimization

Sanjoy Das
Kansas State University, USA

Bijaya K. Panigrahi
Indian Institute of Technology, India

Shyam S. Pattnaik
National Institute of Technical Teachers' Training & Research, India

ABSTRACT

This chapter focuses on the concepts of dominance and Pareto-optimality. It then addresses key issues in applying three basic classes of nature inspired algorithms – evolutionary algorithms, particle swarm optimization, and artificial immune systems, to multi-objective optimization problems. As case studies, the most significant multi-objective algorithm from each class is described in detail. Two of these, NSGA-II and MOPSO, are widely used in engineering optimization, while the others show excellent performances. As hybrid algorithms are becoming increasingly popular in optimization, this chapter includes a brief discussion of hybridization within a multi-objective framework.

INTRODUCTION

Many real world optimization problems cannot be formulated readily as one involving either the minimization or the maximization of a single objective function. Under these circumstances, the concepts of dominance and Pareto-optimality are usually invoked (Deb, 2001; Das & Panigrahi, 2008). A multi-objective framework provides a formal basis to develop effective optimization algorithms, and to evaluate their performances.

Nature inspired algorithms, heuristic approaches that tend to mimic various natural phenomena, have been very successful in addressing multi-objective optimization problems. These algorithms are typically population-based approaches that store a set (or population) of solutions, which are regularly updated – a feature that is significant in a multi-objective framework, as it is not easy to discern between

DOI: 10.4018/978-1-60566-766-9.ch004

a good solution and a bad one. Furthermore, being stochastic approaches these algorithms are equipped to handle local minima in the fitness landscape. Lastly, these methods can be hybridized readily with greedy algorithms for faster convergence (Das, 2008).

Evolutionary algorithms are a popular scheme for nature-inspired multi-objective optimization (Goldberg, 1989; Mitchell, 1998). These algorithms are based on Darwinian mechanisms of natural selection. A recent nature inspired approach is particle swarm optimization (PSO). PSO borrows from swarm intelligence, where simple interactions between individuals (called particles) enable the swarm to converge to optimal locations in the search space (Clerc, 2005). Yet another widely used widely used natural paradigm for multi-objective optimization is based on artificial immune systems (AIS) (de Castro & Von Zuben, 2002).

The multi-objective optimization has been applied to train a wide variety of machine learning models. Usually, in these cases, the training error is treated as only one of the objectives for optimization, while other aspects of the learning system, such as regularization features, network size, or number of kernels, are also considered as other objectives.

It must be noted here that this chapter deals with continuous optimization problems. Combinatorial optimization problems are another class of problems, for which multi-objective algorithms exist (*e.g.* multi-objective ant colony optimization). However, such studies are not as extensive as in case of continuous optimization. Moreover, the usefulness of combinatorial optimization algorithms in machine learning is rather limited. Therefore, combinatorial optimization shall not be addressed in this chapter. Needless to say, the nature inspired approaches discussed here can be modified for combinatorial optimization problems.

BASIC CONCEPTS OF MULTI-OBJECTIVE OPTIMIZATION

Let the search space of the multi-objective problem (*i.e.* the space of all solutions) be denoted as $\Sigma \subseteq \Re^N$, where N is the dimension of each solution vector. Without loss of generality, it shall be assumed that the multi-objective optimization problem involves the simultaneous minimization of M objectives, $f_1, f_2, \ldots f_M$. The objective function space $\Omega \subseteq \Re^M$ is defined as the set $\left\{\left(f_1(\mathbf{x})\ f_2(\mathbf{x}) \ldots f_M(\mathbf{x})\right) \middle| \mathbf{x} \in \Omega\right\}$. It should be noted that typically, the dimensionality of the objective function space, M, is much lower than that of the search space, N. The relationship between the search space and the objective function space is illustrated in Figure 1. Below, for a simple case where $N = 3$ and $M = 2$.

In the same figure are also shown two solutions, $\mathbf{x}$ and $\mathbf{y}$ in Σ, as well as their images in Ω. From the latter, it is clear that $\mathbf{x}$ is a better solution than $\mathbf{y}$, as $f_1(\mathbf{x}) < f_1(\mathbf{y})$ and $f_2(\mathbf{x}) < f_2(\mathbf{y})$. We say that $\mathbf{x}$*dominates*$\mathbf{y}$. Mathematically this relationship is expressed as $\mathbf{x} \prec \mathbf{y}$. When M objectives are involved, we say that a solution, $\mathbf{x}$ dominates another one, $\mathbf{y}$ if and only if $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$ for $i = 1, 2, \ldots, M$, with the inequality being strict for at least one objective, *i.e.*, $f_i(\mathbf{x}) < f_i(\mathbf{y})$ for some i. In any population of solutions $\mathbf{P}$, the set of all solutions that are not dominated by any other solution is called the *non-dominated set* $\Phi(\mathbf{P})$. Mathematically we write,

$$\Phi(\mathbf{P}) = \left\{\mathbf{x} \in \mathbf{P} \middle| \text{ for all } \mathbf{y} \in \mathbf{P},\ \mathbf{y} \prec \mathbf{x} \text{ is false}\right\}. \tag{1}$$

Related Content