Chapter 6
Solving IVP with ODE Solver Commands

ABSTRACT

The chapter presents solvers of initial value problems (IVP) of an ordinary differential equation (ODE). The steps required to solve the explicit and implicit differential-algebraic equations (DAE) using available MATLAB® ode-solvers are described in detail. The solutions are presented through real examples. In the final part, the studied ode-solver commands are used to solve various engineering problems. These include the IVP ODE solutions for second order electrical circuit, cantilever beam deflection, relative strength of the opposing military forces, atmospheric behavior model, and kinetic-type DAE system, as well as some others.

6.1. INTRODUCTION

Various processes and phenomena are described in the natural science by differential equations (DEs), therefore they are studied in universities, used by students, graduates, engineers, and actually occupy a central place in science, engineering, and technology. Accordingly, a tool for solving DEs is included in the basic MATLAB® package. Due to the fact that even ordinary DEs are often impossible to solve analytically, the only possibility is a numerical solution. As you may have seen in the previous chapter, there is no universal numerical method, and the basic MATLAB® provides special commands, called *solvers*. Solvers designed for ODEs are divided into two groups depending on the problems they solve: initial value problem (IVP) solvers and boundary value problem (BVP) solvers. When the initial value problem is solved, the initial value of the search function is specified, at some point in the solution range (termed the domain). Description of the ode-solvers for the initial value problem is presented in this chapter. The material is provided with the following application examples: second order electrical circuit, cantilever beam deflection, relative strength of the opposing military forces, atmospheric behavior model, and kinetic-type DAE system.

The following assumes that the reader has at least a cursory familiarity with the material presented in the previous chapters.

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6.2. SOLVERS FOR ODEs AND DAEs

MATLAB® ODE solvers are designed to solve explicit ODEs $\frac{dy}{dt}=f\left(t,y\right)$, implicit ODEs $f\left(t,y,\frac{dy}{dt}\right)=0$, and differential algebraic equations (DAEs); the latter are systems of ODEs that include equation/s that do not contain differential terms $\frac{dy}{dt}$. The following describes how to use the available software tools.

6.2.1. Required Form of the Explicit ODEs

The solver designed to solve a first order equation written in form $\frac{dy}{dt}=f\left(t,y\right)$. To solve an *n*-order ODE $\frac{d^n y}{dt^n}=f\left(t,y,\frac{dy}{ddt},\frac{d^2 y}{dt^2},\ldots,\frac{d^{n-1}y}{dt^{n-1}}\right)$ using a command of the ODE solver the equation should be represented as a system of the *n* first-order DEs:

$$\left.\begin{aligned}
&\frac{dy_1}{dt}=f_1\left(t,y_1,y_2,\ldots,y_n\right)\\
&\frac{dy_2}{dt}=f_2\left(t,y_1,y_2,\ldots,y_n\right)\\
&\ldots\\
&\frac{dy_n}{dt}=f_n\left(t,y_1,y_2,\ldots,y_n\right)
\end{aligned}\right\} \tag{6.1}$$

where y_1, y_2, …, y_n are the dependent variables while *t* is the independent variable varying between starting t_s and final t_f values; note that names of the dependent and independent variables are arbitraries, e.g. *x* can be used instead of *t*, or *u* instead of *y*, etc.

To reduce high-order ODE to a set of first order ODEs (6.1), a replacement method is applied that uses the substitution $\frac{dy_i}{dt}=y_{i+1}$ is used. Explain this for a second order ODE $\frac{d^2 y}{dt^2}=f\left(t,y,\frac{dy}{dt}\right)$. This equation can be reduced to two first order ODEs as follows: denote *y* as y_1, then denote the first derivative $\frac{dy_1}{dt}$ as y_2 and substitute y_2 into the original equation $\frac{d^2 y}{dt^2}=\frac{d}{dt}\left(\frac{dy_1}{dt}\right)=\frac{dy_2}{dt}=f\left(t,y_1,y_2\right)$. Thus, as a result, you obtain a system of two ODEs of the first-order

$$\begin{aligned}
&\frac{dy_1}{dt}=y_2\\
&\frac{dy_2}{dt}=f\left(t,y_1,y_2\right)
\end{aligned}$$