Chapter 1
Introductory Remarks

ABSTRACT

This chapter provides introductory remarks about the book objectives, audience and order of use of the material as well as presents the essentials of the ordinary and partial differential equations (ODE and PDE) solvable using the dedicated solvers available in the MATLAB® base package. In this regard, the classification of the ODEs is briefly explained along with descriptions of the initial and boundary value problems. A classification of PDEs and associated initial and various types of boundary conditions are also presented.

1. 1. INTRODUCTION

In science and in many engineering fields, ordinal and partial differential equations, ODEs and PDEs, play a key role as they describe technological phenomenon and processes and are vital for the analysis, design and modeling of optimal technical products. MATLAB® is introduced in the book as an essential foundation for ODE and 1D PDE solvers and programmatic tool of the Symbolic Math ToolboxTM with appropriate explanatory solutions applied to both traditional and emerging engineering problems.

Therefore, when starting to study the material of the book, it is useful to recall the basic concepts and fundamentals of differential equations, DEs. Some key points regarding the classification of DE are summarized below to help the reader understand and practically use DE solvers in solving their current problems. In this chapter, the classification of ODEs is presented and the associated initial value and boundary value problems, IVP and BVP, are described. Along with this, the classification of PDEs is given and possible Dirichlet, Neuman, and Robin boundary conditions, as well as available initial conditions are discussed.

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1.2. CLASSIFICATION OF THE ODEs

Here are some mathematical definitions accepted for ordinary differential equations (see, for example, Polyanin & Zaitsev, 2017; Agrawal & O'Regan, 2008).

Let t and y be the independent and dependent variable, respectively. An equation that includes these variables and various derivatives of y is called a differential equation (DE). The following are some examples of ordinary DEs (or, shortly, ODEs):

$$\frac{dy}{dt} = 2ty + e^{-y}$$
$$\frac{d^2y}{dt^2} = \left(\frac{dy}{dt}\right)^2 + \sin 2\pi t$$
$$\frac{d^3y}{dt^3} - 3\left(\frac{dy}{dt}\right)^4 = y^2$$

The general form of ODE is written as follows:

$$F\left(t, y, \frac{dy}{dt}, \frac{d^2y}{dt^2}, \ldots, \frac{d^ny}{dt^n}\right) = 0$$

The ODE contains only one independent variable and various order derivatives with respect to this single variable.

The order of the highest derivative determines the order of the differential equation. So, the first, second and third equations of the above example are first, second and third order ODEs, respectively.

The highest power of the highest order derivative determines the degree of the ODE. So, all three equations in the first example are the first degree because their higher derivatives have a first-degree power.

If y and its derivatives $\frac{dy}{dt}, \frac{d^2y}{dt^2}, \ldots$ are linear in y, the equation is called linear; otherwise, it is called nonlinear. The general form of a linear ODE is expressed as

$$c_0 \frac{d^ny}{dt^n} + c_1 \frac{d^{n-1}y}{dt^{n-1}} + \ldots + c_{n-1} \frac{dy}{dt} + c_n y = f\left(t\right) \tag{1.1}$$

where $c_0, \ldots, c_n$ are constants and $f(t)$ is a function of t.

The following are examples of linear ODEs: