

Chapter 10

Matrix Algebra and System of Linear Equations

ABSTRACT

This chapter deals with an elementary introduction to matrix algebra. First, we give a brief introduction to vectors and vector operations, then we discuss basic materials on matrices, determinants, their properties and the solution of linear system of equations. We also consider the solution of the systems of linear equations by using the inverse of the coefficient matrix of the given system of equations. We introduce the Gaussian method based on a series of algebraic operations which are compatible with computer software. Regarding the importance of eigenvalues, the eigenvalues and eigenvectors, which are numbers and vectors associated with square matrices. The last section of this chapter considers some applications of matrices to several examples we encounter in industry or applied mathematics.

10.1 INTRODUCTION

The matrices play a very important role in practical issues and are widely used in various fields of science and engineering. The general purpose of this chapter is to familiarize with the basic concepts and definitions of matrices and related algebraic operations and to define the determinants of a matrix and its related features. We start with a simple example. Suppose we want to specify a polynomial of the degree 2, i.e., $P(x)=ax^2+bx+c$ so that, $P(1) = 2, P'(1) = 0$ and $P''(1) = -2$. According to this data, we have,

$$P(1) = a + b + c = 2$$

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$$P'(1) = 2a + b = 0$$

$$P''(1) = 2a = -2.$$

From the third equation, we get $a=-1$. We substitute it in the second equation, we obtain $b=2$. If we put the values of a and b in the first equation, we will have $c=-1$. Therefore, we could simply solve this system of equations. Nevertheless, we are going to introduce some materials for solving such a system with other methods.

From these equations, we can have the following arrangements.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}.$$

Each of these arrangements is called a *matrix*, which consists of several rows and columns. Two of these matrices, which have only one column, are formally called *vectors*.

The main subject of this chapter devoted to the concept of a matrix and matrix algebra will enable us to consider the solution of such a system of linear equations more systematically and to express the result more compactly. In this chapter, our main goal is to consider two fundamental problems, which are: (i)-The solution of a linear system. (2)- The calculation of eigenvalues and eigenvectors of a matrix. But first, we mention a brief introduction to vectors which may be useful in this book. In general, a quantity that is defined by both magnitude and direction is a vector. Many physical quantities such as force, speed, and acceleration are vector quantities.

By a vector $\mathbf{a}$, we mean a list of numbers which are called the *components* of $\mathbf{a}$. That is, $\mathbf{a} = (a_1, a_2, \dots, a_n)$.

The length or *norm* of $\mathbf{a}$ denoted by $\|\mathbf{a}\|$ is a positive quantity defined by

$$\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}.$$

A vector with length unity is called a *unit vector*.

We usually represent a vector with three components a_1 , a_2 and a_3 in terms of three unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ having the directions of the positive x , y and z axes of a three- dimensional rectangular coordinate system and denoted by

$$\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}.$$

From the following figure, vectors $\mathbf{OC}$ and $\mathbf{OM}$ can be denoted by:

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