Chapter 8
Complex Numbers and Polar Coordinates

ABSTRACT

In this chapter, we introduce complex numbers, which are a generalization of real numbers. While explaining some of their applications, we examine the solution of the equations that may have complex roots or complex coefficients by introducing the polar form of a complex number. We will see how the equations of some curves in the polar form are simpler than the Cartesian form. In particular, they make the solution to a given problem easier. Since in practice we encounter certain types of curves in polar form, hence we also consider polar forms of some specific curves and then obtain some formula for calculating the area of a bounded region. We also, like in previous chapter, add Computer Station to help readers to draw the curves in polar forms.

8.1 INTRODUCTION

For a long time, the set of real numbers was considered as the set of whole numbers, but in the early 16th century, when there was a great desire to solve algebraic equations, it was impossible to solve, even, a very simple algebraic equation like $x^2+1=0$. For example, suppose we want to find two numbers whose sum and product are 2. If we assume two numbers are x and y, then we have to solve the following system to find these two numbers.

$$\begin{cases} x + y = 2 \\ xy = 2. \end{cases}$$

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But by eliminating y from these two equations we get, $x^2+2x+2=0$, from which the solutions will be, $x = 1 \pm \sqrt{-1}$. Mathematicians at that time found that answers like $x = 1 \pm \sqrt{-1}$ do not belong to the set of real numbers because the square of every real number is positive. For this reason, the generalization of the set of real numbers to another set that gives such an answer has become stronger. Fortunately, at the time of Descartes, these numbers, which today are called *imaginary numbers*, were accepted as part of a system of numbers, but Leonardo Euler, a French mathematician, used the sign i to represent, $\sqrt{-1}$ and therefore, the values of $\pm i$ were attributed to the equation, $x^2+1=0$. You can look at 2 as an imaginary number. It doesn't matter what this imaginary number can be, the important thing is that this seemingly imaginary number solves a problem of this type and guides us to a new world of mathematical realities. Something that guides us to reality must be a reality.

8.2 BASIC CONCEPTS AND DEFINITIONS OF COMPLEX NUMBERS

An expression of the form $z=x+iy$ in which $x,y \in \mathbb{R}$ and $i = \sqrt{-1}$ (with, $i^2=-1$) is called a *complex number*. We denote the set of all complex numbers by ¢. Therefore,

$$¢ = \{z = x + iy | x,y \in \mathbb{R},\ i^2 = -1\}.$$

We call x and y the real and imaginary parts of z, respectively, in symbols, $x=Rex, y=Imz$. Then a complex number is an ordered pair of real numbers x and y. Thus, we can write $z=(x,y)$. If $x=0$, we call the number z a *pure imaginary number*.

There is no order property in complex numbers. For example, it cannot be said that the number z_1 is bigger or smaller than the number, z_2. But two numbers $z_1=x_1+iy_1$, and z_2-x_2+iy_2 are said to be equal, if and only if they have equal real parts and the same imaginary parts. That is,

$$z_1 = z_2 \leftrightarrow x_1 = x_2, y_1 = y_2.$$

On the set of complex numbers, operations such as addition, subtraction, multiplication, and division can be defined.

Addition and subtraction of two numbers, $z_1=x_1+iy_1$ and z_2-x_2+iy_2 is defined by:

$$z_1 \pm z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2). \tag{1}$$

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