Chapter 6
Integral Calculus

ABSTRACT

This chapter is devoted to integral calculus. This includes two different integrals, namely definite and indefinite integrals. After finding integration of several elementary functions that we met in Chapter 3, we introduce some methods for integrating functions that have wrapped forms. Various applications of integration will be given throughout this chapter in particular, area, volume of a bounded area and length of an arc of a curve. We also call the Mean Values Theorems as an optional part for integration. We define improper integrals which can be used in finance and the continuous random variables in Chapter 12. For evaluating some definite integrals which cannot be evaluated by methods discussed in calculus, we described two numerical methods for such a task.

6.1 INTRODUCTION

In differential calculus, it was intended to investigate the processes of achieving the rate of change of a dependent variable, but in many cases, the rate of change may be certain, and the problem is to find the main value of this variable. This process is called integral calculus, which is the reverse of differential calculus.

Integral calculus includes the study of two different forms of integral called *definite* and *indefinite integrals*. When the general answer to the problem is desired, the indefinite integral is used, while when the problem is for specific limits or a certain range of values, the definite integral is used. We start our work with indefinite integrals and then, when we know the methods of integration to some extent, we will examine the definite integral and its applications. But firstly, we mention the definitions and fundamental theorems related to integral calculus.

DOI: 10.4018/979-8-3693-3657-1.ch006

6.2 FUNDAMENTAL CONCEPTS OF INTEGRAL

Definition 1: The function $F(x)$ is called *antiderivative* or *the primitive function* of $f(x)$ in the interval I if all x belong to this interval, we have $F'(x)=f(x)$

For example, $F(x)=x^2$, is a primitive function of $f(x)=2x$, or $F(x)=\sin x$, is a primitive function of $f(x)=\cos x$,

Since the derivative of $F(x)+c$ is also $f(x)$, then in a more general case, the primitive function of $f(x)$ is, $F(x)+c$.

According to the definition of the primitive function, it can be deduced that the operation of finding the primitive function is a kind of *antiderivative* operation. When the operation of division is the reverse of the operation of multiplication or $\sin^{-1}$ is considered as the reverse of operation of sin operation, we expect to know, what is the reverse of the differential operation? Suppose, $F(x)+c$ is the primitive function of $f(x)$ then

$$d(F(x)+c) = F'(x)dx = f(x)dx,$$

which leads to finding a function such that its differential is $f(x)dx$. That is,

$$f(x)dx = d(F(x) + c).$$

Definition 2: The reverse operation of differential, denoted by "$\int$" is called *integral*.

With this definition, if we apply the operation $\int$ on the sides of the above equality, we get

$$\int f(x)dx = F(x) + c. \tag{1}$$

The function $F(x)+c$ is called the indefinite integral of the function, $f(x)dx$. The component $f(x)$ is called the *integrand* of integral, and $d(x)$ indicates that the integration is achieved with respect to x. We often use the symbol $\int f(x)dx$ to denote any indefinite integral of $f(x)$.

Because integration is the reverse of differentiation, theorems related to integration can be obtained from differential theorems. Therefore, the following theorems can be easily proved by using the corresponding theorems in differential calculus.

Theorem 1. If the functions $f(x)$ and $g(x)$ are integrable and k is a constant number then, the following rules hold:

(i) $\int[f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

(ii) $\int kf(x)dx = k\int f(x)dx$

(iii) $\int[f(a(x))\,g(x)]dx = F(G(x)) + c,$

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