Chapter 5
Differential Calculus

ABSTRACT

In this chapter, we deal with differential calculus. By definition of differentiation, its interpretations we will derive the differentiations of several functions which are defined in Chapter 3. Besides finding tangent and normal lines, some more applications of differentiation will be given throughout the chapter. Another interpretation of the derivative, which is the instantaneous speed that is observed in physics, will also be considered here and give some examples of its application. After introducing the concept of differential, we introduce chain rules and parametric differentiation. The concepts maxima and minima for functions will be described. We will state Roll's, Lagrange's and Cauchy's theorems as an optional part and then introduce Taylor's formula, which is the most useful one in calculus.

INTRODUCTION

One of the important concepts in differential calculus that is used in many practical and theoretical fields is the concept of derivatives. Most of the relationships between the quantities of the physical world can be expressed using continuous and differentiable functions.

We know that in the function $f{:}A \rightarrow B$ when the variable x in A, changes, depending on that variable, the function y in B also changes based on the rule $y=f(x)$. One of the important topics of the function is to determine the number of y changes compared to x changes at any point. This goal is fulfilled by the derivative.

The derivative is one of the basic concepts of mathematics that has many applications in all branches of science. The concept of speed and acceleration of a moving object, the heating and cooling of an object, the reaction speed of a chemical process, the rate of decay of a radioactive process and the growth of a biological organism are among the derivative applications.

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Derivatives also play a crucial role in managing financial risks, optimizing investment returns, and informing strategic decision-making in the field of management.

5.2 CONCEPT OF DERIVATIVE AND ITS INTERPRETATIONS

Suppose the real function f is defined on the interval (a, b). For $x \in (a,b)$, we form a function of the new variable h with the rule $\frac{f(x+h)-f(x)}{h}$. The limit of this function is called the *derivative* of the function f at the point x when $h \to 0$ (if there is a limit) and is denoted by $f'(x)$. Thus

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad (1)$$

We call h the *increment* of the independent variable and $\Delta f = f(x+h) - f(x)$ the *increment of the function*. Then, another definition formula for differentiation is

$$f'(x) = \lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x} \qquad (2)$$

The derivative of a function according to the above definition, if it exists, is a real number. If for all the points, in (a,b), there exists a derivative of f then it is said that f has a derivative in (a,b). In this case, a function is defined on (a,b) that for each point $x \in (a,b)$, corresponds to the derivative of f at that point, i.e., $f'(x)$. This function is called the *derivative function* and is denoted by f'. If a function has a derivative at each point $x \in (a,b)$, it is said to be *differentiable* in this interval.

Example 1: Verify that the function $f(x)=x^2$ has a derivative at every point of $\mathbb{R}$. Find its derivative.

Solution: The function $f(x)=x^2$ is continuous on $\mathbb{R}$, and we also have

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}\frac{2hx}{h} = 2x$$

Thus, f' exists on $\mathbb{R}$, and $f'(x=2x$.

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