Chapter 3
Functions

ABSTRACT

This chapter deals more with functions. First, we introduce a Cartesian product and its properties, then define the concept of a relation which is a subset of the Cartesian product of two sets. By defining the concept of function, we introduce dependent and independent variables and domain and range of a function that a function can accept. After introducing two specific types of functions, we are going to be familiar with several functions and their properties throughout this chapter. We also define the inverse of a function, and then we introduce and draw figures of most of them to have more ideas about such functions.

3.1 INTRODUCTION

When we formulate a natural process in mathematical language, we will undoubtedly have a relationship between its members. In mathematics, the word relation is a common term used to indicate a relationship between two members of a set. Relationships occur everywhere. For example, when we say that this person is an employee of Company C, we have established a relationship between the employee and the company. In a database where the data is arranged alphabetically, their relationship is such that one precedes the other. In general, when we say A is a subset of B, we establish a kind of relationship between A and B.

Using the theory of sets, we can present a special type of relationship, which is known as a function, and then introduce several types of them, many of which are used in practical management problems.

In the previous chapter, we learned about three operations, namely, union, intersection and difference of sets. In this chapter, we will get acquainted with another operation called a *Cartesian product* between two sets and use it to define *relations* and *functions*.

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Functional relationships are related to a specific set and any desired relationship for both selected members must belong to this specific set. We consider these two members as a special pair of this set.

3.2 CARTESIAN PRODUCT

We know that both objects such as x and y form a pair, and if order is considered for them, it is called an *ordered pair* and is represented by (x,y), therefore, $(x,y) \neq (y,x)$. Two ordered pairs are equal when their correspondence components are equal.

We can introduce the ordered 3-tuple (x,y,z), or generalize it as the ordered n-tuple, but here we will deal with the ordered pair (x,y).

The *Cartesian product* of two sets A and B, denoted $A \times B$, is the sets of all ordered pairs of the form (x,y) where $x \in A$ and $y \in B$. According to this definition, $A \times B$ is a set of ordered pairs whose first component belongs to A and the second component belongs to B. For example, if $A = \{a,b\}$ and $B = \{b,c,d\}$ then

$$A \times B = \{(a,b),(a,c),(a,d),(b,b),(b,c),(b,d)\}.$$

It is not difficult to show that $A \times B \neq B \times A$. That is, the Cartesian product does not have commutative property.

It is clear that if A has m members and B has n members, then $A \times B$ has $m \times n$ members.

The Cartesian product has the following properties:

(i) $A \times \varnothing = \varnothing \times A = \varnothing$
(ii) $A \times B = B \times A \leftrightarrow A = B$
(iii) $n(A \times B) = n(B \times A)$
(iv) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(v) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(vi) $A \times (B - C) = (A \times B) - (A \times C)$
(vii) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

The set $A \times B$ can be represented as a set of plane points called the *Cartesian coordinate plane*. In other words, if

$$A \times B = \{(x,y) | x \in B \wedge x \in A\},$$

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