Chapter 8
Ito's Calculus for Stock Price Prediction for the Johannesburg Stock Exchange Market

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ABSTRACT

The Johannesburg Stock Exchange is positioned as the leading African stock market and listed among the top twenty in the world, considering market capitalization. Price prediction remains at the center of concerns for stock markets in Africa and the world. Ito's calculus has extended the calculation methods to stochastic processes, Brownian motion, and stochastic differential equations with some relevant applica-

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tions in mathematical finance. This work aims to present a rigorous mathematical justification for the models used by financial practitioners for pricing derivatives and managing risk. Additionally, this research will develop mathematical models, background of pricing models with comprehensive proofs, and test their reliability with historical data collected from the Johannesburg Stock Exchange for future decision-making.

INTRODUCTION

Kiyosi Itô, a Japanese mathematician, introduced the so-called Itô calculations, which extended the stochastic processes of ordinary calculation methods. His work revolutionized financial mathematics and the resolution of stochastic differential equations.

Starting from the Riemann-Stieltjes integral, whose components he was supposed to be stochastic processes, Itô introduced a stochastic generalization. Knowing that ordinary calculation methods cannot be applied to Brownian motions, an integral from 0 to t that considers Itô's conditions remains defined only when the integrand is adapted. In other words, the Riemann sum is constructed following the choice of a sequence of partitions of $[0, t]$ into N subintervals. Knowing which point is indicated for calculating the sum is imperative for each subinterval. The limit presented in Definition 1 makes it possible to respond to this concern. The proof of existence and independence from any sequence f_n chosen, of this limit, calls upon many mathematical techniques.

This so-called stochastic generalization of the Riemann Stieltjes integral includes a change of variables called Itô's Lemma and the integration by parts formula. Itô's Lemma, as presented in dimension one in equations (4) for the integral form and (5) for the differential form, has the particularity of having quadratic variation terms. Following this advantage and its other properties, Itô's Lemma remains one of the most indicated tools in modeling price movements in financial markets.

LITERATURE REVIEW

Numerical approximation methods have been developed to decipher the solutions of stochastic differential equations, particularly if analytical solutions are not always obvious. Some relevant work on pricing has been done using Itô's Calculus. It was used by Xuerong Mao et al. in 2005 to develop several new techniques to solve mathematical problems from the non-Lipschitz coefficients and the regime-switching. They have used the Euler-Maruyama method as an approximation method. (Mao