Chapter 7
Fixed Point Theory and Insurance Loss Modeling:
An Unlikely Pairing

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ABSTRACT

This study focuses on the future development of an insurance company during difficult circumstances, which can be described by a stochastic process that must be effectively managed to achieve the best goals for the company. Effective risk or loss management models can bring in more revenue for the insurer and result in less conditional pay-out of claims to the insured. While insurance losses, risks, and premium calculation are important topics in the field, existing literature has not always stood the test of time due to the dynamic nature of insurance principles and practices. There is a need for a suitable loss model that can adjust loss rating to a particular experience and provide an appropriate and equitable premium. The aim of this research is to find sufficient conditions for the convergence of an algorithm towards a fixed point under typical insurance loss and actuarial circumstances, resulting in a uniquely determined solution. The study presents a unique fixed point, which the algorithm converges towards through straightforward and simplified generalised formulae and functions.

1. BACKGROUND TO THE STUDY

The future development when an insurance company is in a difficult circumstance can be described by a stochastic process which the insurance company is tasked to manage effectively in order to achieve best goal of the company. Fixed point theorem is very general but we shall narrow its application to insurance business problem to get precise formulation. In studying some nonlinear phenomena, fixed point theorem is an important and powerful tool that can be applied in many fields. This research paper attempts to apply fixed point theorem in the areas of insurance business just as it has been applied to

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geometry, analysis, number theory, set theory, group theory, algebra, dynamics, topology and so on. It is noteworthy to explain briefly what a fixed point theory means. According to Rajic, Azdejkovic and Lonar (2014), fixed point theorem concerns itself with the examination of the existence of a certain point, say y, in the domain of a function, say g, where $g(y)=y$. The identical function mapping and the function values are equal. This means that any marginal change in the function of y will proportionally result to additional fixed points. If $g(y) = y$ then $g(y) - y = 0$. Therefore if a certain function f is shown as $f(y) = g(y) - y$, function g has zero as the fixed point. If Y is a set and $g: Y \rightarrow Y$ is a map from Y to Y, a point $y \in Y$ is known as a fixed point of g since $g(y) = y$. For a family of G of Y, G is a semigroup or group. Here the fixed point theorem gives specific condition on Y and G ensures that there exists a simultaneous fixed point in $y \in Y$ for $g \in G$. If G is a group F, it arises from a group action $\Omega: F*Y \rightarrow Y$ of F on Y. If $\Omega(F, Y) := fy$, it is assumed that 1 of F is the identity of Y. Also, if $a \in F$ and $y \in Y$ for all f such that $(fa)y = f(ay)$ such $y = F$-space, Y = topological space, F = topological ground and Ω is jointly continuous. If there exists an action group F on x where Y(power set$)=P(x)$, it leads to an action of F on $P(x)$ and the F-invariant (subset of X) is the fixed point of the new action. In other words, the fixed point theorem leads to an F-invariant set. Under the existence of Haar measure Ψ on a compact topological group F, left invariance indicates that Ψ is F- fixed due to the action of F by left translation on the space $N^+(F)$ of any finite positive measure on F. If Ψ is normalised and finite so that $\Psi(F)= 1$, it can be referred to as F-fixed point under the action of F on the convex set of normalised measure on F.

The concept of fixed point theorem finds its application in various fields including insurance. The use of fixed point theorem in insurance companies has become a major topic in actuarial science as it helps to develop appropriate pricing models for insurance risks. Insurance companies often make use of statistical methods to estimate the likelihood of a loss occurring and the magnitude of such loss. This is done through the use of historical data, such as claims experience and premium income, to predict future outcomes. However, relying solely on statistical models can be insufficient as these models are often limited by their assumptions and predictive power. Therefore, it is essential to employ fixed point theorem in the pricing models of insurance risks.

Fixed point theorem is particularly useful in situations where there are uncertainties and complexities in the modeling of insurance risks. As mentioned earlier, a qualified actuary in an insurance company must make use of an appropriate insurance loss model to price a risk. By applying fixed point theorem, the actuary can determine the existence of a fixed point in the domain of the loss function, where the function values are equal (Belzunce et al., 2016). This enables the actuary to determine an appropriate premium that covers the cost of the risk while ensuring that the insurer is not at risk of ruin.

In order to apply fixed point theorem in insurance risk modeling, the actuary must first identify the relevant parameters and the underlying distribution of the risk. The actuary can then construct an appropriate loss function, which represents the expected loss incurred by the insurer for a given level of risk exposure. The actuary can then apply the fixed point theorem to determine the existence of a unique solution to the loss function. This unique solution represents the equilibrium point at which the insurer can price the risk to cover the expected losses without incurring excessive costs.

It is important to note that the application of fixed point theorem in insurance risk modeling requires a deep understanding of the underlying assumptions and uncertainties of the model. The actuary must also have a sound knowledge of statistical methods and mathematical modeling techniques. Furthermore, the actuary must be able to interpret and communicate the results of the fixed point theorem analysis to stakeholders, including management and regulators.

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