

# Chapter 3

## A Discussion on Non-Convex Optimization Problems Arising in Supply Chain Design and Finance

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### ABSTRACT

*Non-convex optimization problems belong to a class of classical nonlinear optimization problems, which are often difficult to solve. An optimization problem becomes non-convex due to the presence of non-convex functions in the objective function or constraints. A function is a convex function if its Hessian matrix is positive and semi-definite for all values; otherwise, it is a non-convex function. A Hessian matrix is called positive semi-definite when the eigenvalues of the matrix are non-negative. A non-convex function can be either a concave function or a function that is neither a concave nor a convex function. A concave function is always negative semi-definite, indicating that the eigenvalues of the matrix are non-positive. This chapter starts with a short introduction to non-convex problems, followed by a discussion on different non-convex problems arising in supply chain and finance. Thereafter, the authors discuss different algorithms used for solving non-convex problems. Finally, the chapter concludes with the limitations of different algorithms.*

### INTRODUCTION

Non-convex optimization problems belong to a class of classical nonlinear optimization problems, which are often difficult to solve. Non-convex problems have been of interest for the last eighty years. An optimization problem becomes non-convex due to the presence of non-convex functions in the objective function or constraints. A non-convex function can be either a concave function or a function that is neither concave nor convex. Let's consider a non-convex optimization problem of the following kind:

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$$\min_x f(x) + \phi(x) \tag{1.1}$$

$$\text{Subject to } g_i(x) \leq 0, \forall i=1,2,\dots,I \tag{1.2}$$

$$x_k^l \leq x_k \leq x_k^u \forall k = 1, \dots, n. \tag{1.3}$$

where  $g_i(x)$  and  $f(x)$  are convex functions,  $\phi(x)$  is either a non-convex function. Constraints (1.3) set the bounds for the variable  $x_k$  which may be continuous or discrete. Next, we will briefly discuss the properties of non-convex functions.

### NON-CONVEX FUNCTION

A function is non-convex when the function does not act as a convex function. A function is a convex function if and only if its Hessian matrix is positive semi-definite for all values; otherwise, it is a non-convex function. A hessian matrix is called positive semi-definite if and only if all eigenvalues of the matrix are non-negative. For example, consider a function of  $n$  variables  $f(x)$  defined on a convex set  $D$ .  $f(x)$  is said to be a convex function when the following conditions hold.

- For any two points  $x^1$  and  $x^2$  and  $0 \leq \lambda \leq 1, f[x^1\lambda + x^2(1-\lambda)] \leq f(x^1)\lambda + f(x^2)(1-\lambda)$ .
- The hessian matrix of  $f(x)$  is positive semi-definite for all values of  $x_1, \dots, x_n$ .

Figure 1a shows an example of convex functions. A non-convex function can be either a concave function or a function that is neither a concave nor a convex function. A function is a concave function if its hessian matrix is negative semi-definite, which happens only when all eigenvalues of the matrix are non-positive. For example, consider a concave function of  $n$  variables  $\phi(x)$  defined on a convex set  $D$ .  $\phi(x)$  is said to be concave when the following conditions hold.

- For any two points  $x^1$  and  $x^2$  and  $0 \leq \lambda \leq 1, 0 \leq \lambda \leq 1, f[x^1\lambda + x^2(1-\lambda)] \geq f(x^1)\lambda + f(x^2)(1-\lambda)$ .
- The hessian matrix of  $f(x)$  is negative semi-definite for all values of  $x_1, \dots, x_n$ .

An example of a non-convex function, which is neither concave nor convex, includes an inverse S-shaped function. An inverse S-shaped function is concave in nature initially up to a certain point above which the function changes its shape to convex. The point is often referred as a deflection point or economic point. In Figure 1.1b, we show an inverse S-shaped function. Historically, non-convex optimization problems have been of interest due to several real-world applications. Next, we briefly describe some of the non-convex application problems.

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