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ITP5092

The Effects of Live Problem Solving in an Internet Based Mathematics Tutorial System

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ABSTRACT

Mathematics is an interactive domain where students have to evaluate and produce expressions. During learning the evaluation process is cognitively taxing so it hinders the learning process which is to help students acquire the ability to analyze the given data to find behavioral patterns. Cognitive tools emerged to assist students by performing operations like addition and subtraction for students so that they can focus on their learning goals (van Jooligan, 1999). On the other hand, modeling components attempt to identify types of student errors to better focus on them. This paper presents an Interactive Mathematical Tutorial System that performs complex calculations based on student inputs in addition to having a Mirror Modeler (2002) which mimics the process students follow when they solve questions in order to reproduce their errors. Results show that interaction with a mathematical tutor is highly productive for all problem types while the modeler is only useful in some cases.

INTRODUCTION

The field of mathematics requires students to acquire both arithmetic level knowledge which involves the basic operations such as multiplication, division and the like, as well as to acquire meta arithmetic skills such as to deduce the mathematical expression that is expanded into a given series. For example, to see the series S=9+27+81+243+729 +2187+6561+19683+59049+17747 and realize that $S=S3^m$ from m=2 to m=11. The cognitive skills required here involve assuming different possibilities and then doing the calculations required to expand the expression to check if it gives the required series.

Benjamin Bloom et al. (1956) have designed a taxonomy of cognitive tasks that identifies and orders the level of cognitive functions in the order of increasing complexity as follows:

Taxonomy of Cognitive Levels
(in order of increasing complexity)

Knowledge
Comprehension
Application
Analysis
Synthesis
Evaluation

So students start by "knowing" the definition of the operations in the series form such as multiplication, division, addition, summation and what these imply. Then students should be able to explain these operations and how they work in their own words. At a higher level they should be able to apply their knowledge to expand the series and evaluate it. Analysis is the stage required to identify the similarities and differences between the different numbers in the expanded form in order to be able to deduce the series form that has a summation.

It should be clear that the calculation process is at a lower cognitive level that the one students are targeting and consequently may impose itself as a hindrance to the learning target at hand. So as students are engaged in checking one of the possibilities, making mathematical errors and correcting, them diverts their attention from the characteristics that would accompany each operation. For example, one possible characteristic is that the power series values grow faster than those in a multiplication series. "Cognitive Tools" (van Jooligan, 1999) are defined as any tool that assists learning such that it would explicitly represent the operations required by the calculations and displays them leaving students to concentrate on achieving their learning targets by learning the meta thinking skills required by the task. The simplest form of such a tool is a pen and paper, where students can write notes to relieve their memory from recalling the numbers while the addition or subtraction is performed.

Therefore it should not be surprising that computer based educational systems impose themselves at the top of the list of Cognitive Tools. One of the potential advantages that they may offer is an active learning environment. Lawrence, Badre and Stasko (1994) found that students who are exposed to an interactive lab session where they are allowed to create their own graphs and observe how the algorithms work on those graphs did significantly better than those who only attended a classroom lecture. They also found, that those who participated in the active laboratory session did better than those who were passively shown an animation of the algorithm. An interesting point to make here, is that this improvement was not evident in declarative questions, instead it was only clear in procedural questions making mathematics perfectly suited as a subject.

This indicates that student learning of any procedural topic is highly influenced by the presentation style of the teaching material and whether or not it is well suited to the topic under consideration. However, this does not imply that students are not highly individual in nature, and would benefit from a system that reflects their individuality. This belief resulted in the development of user modeling as a field. It aims at presenting students with the right types of material at the right point in time in the right presentation style (Fischer, 2001). This necessitated the existence of a "model" that describes student characteristics at least with respect to a particular task.

> "Tutor: What is the integral with respect to x of $x^4/ (1 + x^2)$
> Student: $x + x^3/3 + \tan x$
> ***Tutor (thinks: how did she get that?):…"(Self, 1990)***

One of these approaches attempts to delve into the cognitive workings of the student's mind and tries to best explain how the results could be obtained. Some of those who followed this approach are, Martin & Vahn Lehn (1995), Langley, Wogulis & Ohlsson (1990), Ikeda, Kono and Mizoguchi (1993) amongst others. In order to be able to check the accuracy of the student model in representing the student's cognitive characteristics Vahn Lehn and Niu (2001) conducted a study in sensitivity analysis. They arrived at an interesting conclusion that the accuracy of the model, strongly depends on what the student is allowed to apply during the course of study because the system can only detect knowledge that is being applied, not knowledge that they may have. This shows a limitation to a student modeler in that it is unable to delve into the student's cognitive structure to obtain any more information than is available through the interaction itself.

"When a learner is engaged in a discussion about the learner model, he is reflecting upon his domain knowledge and experience re-calling and re-considering ideas of which he is aware."(Dimitrova et al, 2000)

When students see their models, they find themselves at a higher cognitive level that questions their own learning skills and the errors they are making. This externalization process therefore, provides users with the ability to question the assumptions they made about themselves. The existing approaches for involving the learner in the modeling process include open learner models (Paiva and Self, 1995), collaborative student models (Bull et al, 1995) and interactive diagnosis (Dimitrova et al.,2000).

The system presented here contains a Mirror Modeler (Alkhalifa, and AlDallal, 2002) with respect to its ability to teach the various types of series when contrasted to interactive learning of the same operations on a web based tutoring systems.

MIRROR MODELER INTERACTIVE LEARNING ENVIRONMENT

The System was developed using IBM's Java Visual Age for Java, which is an integrated visual development environment that facilitates the generation of complex functions. Its main features include the ability to import Graphical User Interfaces (GUIs) and Java Beans that could be constant throughout several applications. The tool generates java applets as in the case of this project or Java Servlets as is required.

THE PROBLEM: MATHEMATICAL SUMMATION

The Mirror Modeler was set up to teach Mathematical Summation which is a usually challenging topic to students. The summation is usually represented using a Greek symbol, Sigma (å) and represents the process of adding up the terms in a series. For example, the summation of the series:

$$\sum_{N=1}^{6} N = 1 + 2 + 3 + 4 + 5 + 6$$

Teaching can be in two directions; either giving students the summation notation and asking them to expand it giving the numbers on the right, or giving them the numbers on the right and asking them to return the summation notation. The first task would be at the *Application Level* of Cognitive tasks according to Bloom et al. (1956) while the second at the higher *Analysis Level* and consequently is more challenging than the first. The system is composed of an Interactive Tutorial Section, a Test Section, and a Model Comparison Section.

INTERACTIVE TUTORIAL SECTION

The first section of the system is composed of three main parts that introduce students to the concept of mathematical series. The first takes them through three examples where they generate the series from the summation notation. The system is interactive in that it allows students to select some of the variable values and generates the series accordingly whenever possible.

Note that the problem is a complex one composed of three terms and is broken up into several parts that are calculated dynamically. Students are allowed to specify the starting and ending terms indicating the length of the resultant series and to be able to recognize how the series can change based on different starting and ending numbers.

The second part of the tutorial, is composed of seven examples to the more difficult task of extracting the notation from the series. The tutorial includes the steps to first select the starting and ending point followed by finding a common divisor and then the generation of the terms of the series to check that it is correct. It is difficult here to allow for student flexibility because the problems given are set problems so the system plays the part of a teaching tool at this stage even though all calculations are still done online.

The third part follows with a more interactive practice session where students write the summation notation they believe to be the answer and are shown the resulting generated series. They can then compare this series to the original and practice any number of times they wish.

At this stage students can select from the different given notations and are allowed to practice and see the result of each selection. They are also given advice of the probable cause of error based on the error made. The number of options varies from one problem to the next to test for student learning and to expose students to more than one possible option.

TEST SECTION AND MODEL GENERATION

This section is similar to the third part described above in that it has test questions given to students except that here, students are not shown the resulting series so they are not aware of whether or not their answers are correct. Students are showed three problems and they have to fill in several slots with the answers they believe to be true. In a sense, they break up the notation in a starting number, ending number etc. to allow the system to dynamically evaluate their responses. Student responses, are then analyzed using an expert system that was specifically designed based on a field study of possible student errors in this task.

MODEL COMPARISON SECTION

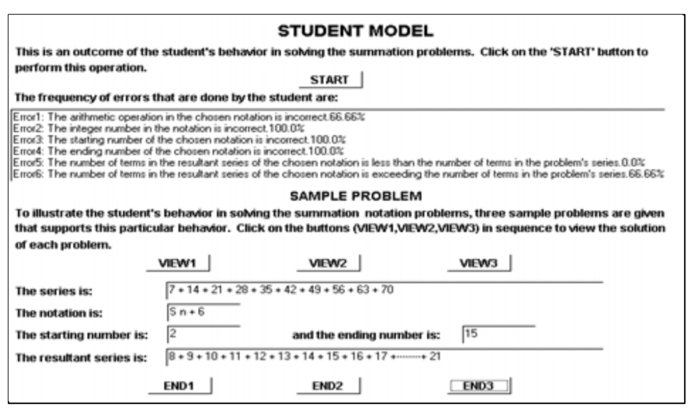
The student modeling component utilizes simple Bayesian rules to extract the probability of that student makes each type of error and it generates a descriptive verbal model with the results. Due to space limitations please refer to the work done in (Alkhalifa, and AlDallal, 2002) for a complete description of the methodology of operation of this component. The types of errors that are detected are as follows:

Error 1 The arithmetic operation in the chosen notation is incorrect.
Error 2 The integer number in the notation is incorrect.
Error 3 The starting number of the chosen notation is incorrect.
Error 4 The ending number of the chosen notation is incorrect.
Error 5 The number of terms in the resultant series of the chosen notation is less than the number of terms in the problem's series.
Error 6 The number of terms in the resultant series of the chosen notation is exceeding the number of terms in the problem's series.

Note that Errors 5 and 6 also depend on Errors 3 and 4, which implies that they are not completely independent and the rules the modeler utilizes reflects that dependence.

The modeler then shows students the ideal solution of each of the sample problems while regenerating how they would solve it using their models as a guide. The idea is to compare their behavior to that of the ideal and allow them to reflect on the causes of their errors.

Figure 1: The Student model with the lower part mimicking student behavior



EXPERIMENT

In order to understand the effectiveness of having an interactive user interface and an open student model, an experiment was performed to evaluate the amount and areas of student learning that occur in a controlled environment. Therefore, an experiment was performed using a pre and post test that are comparable in questions.

Design

21 Students were given a paper and pen test that is composed of three questions that test for the multiplication, power and division operation ahead and following the Interactive Tutorial Section. Results were then compared to those obtained before for 12 students who had the same questions ahead of using the system, and following using the Mirror Modeler. The aim in this experiment is to identify the main learning differences between having both methods available versus having only the Interactive Tutorial Section.

Subjects

21 students from the University of Bahrain participated as volunteers in return for course credit.

Materials

The questions used were specifically selected such that they relate to each other in a way that could be later compared for further analysis. Students would be given the following series of numbers and asked to reproduce the summation Notation that is to the left of each series shown.

Pretest Series

$\sum_{i=1}^{10} i/4$ S=1/4 + 1/2 + 3/4 + 1 + 5/4 + 3/2 + 7/4 + 2 + 9/4 + 5/2

$\sum_{i=1}^{10} 2^i$ S = 2 + 4 + 8 + 16+ 32 + 64 + 128 + 256 + 512 + 1024

$\sum_{i=2}^{11} 3i$ S= 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 + 33

Post-Test Series

$\sum_{m=3}^{12} 11m$ S=33 + 44 + 55 + 66 + 77 + 88 + 110 + 121 + 132

$\sum_{m=2}^{11} 3^m$ S = 9 + 27 + 81 + 243 + 729 + 2187 + 6561 + 19683 + 59049 + 177147

$\sum_{m=1}^{10} m/7$ S= 1/7 + 2/7 + 3/7 + 4/7 + 5/7 + 6/7 + 1 + 8/7 + 9/7 + 10/7

Results

The number of errors produced by question type are 70, 54 and 56 for the pretest questions in the order multiplication, power followed by division. These are contrasted by the number of errors of 25, 28 and 14 for the post test condition. The percentage improvement in each of the question types is 35.7% for the multiplication operation, 20.6% for the power operation and 33.3% for the division operation.

Only significant results of a comparison of the pre and post test according to error type is shown in table 1. The types of errors seem related to the type of series that being learned. Two of the errors; Error 1 which is when students choose the incorrect arithmetic operation and Error 4 which is when students choose a wrong ending number showed significant improvements independent of problem type. Error 3 which is when students get the starting number wrong did not show significant improvement in Question 2 which requires the *power* operation. Error 6 which is that the number of terms in the resulting series exceeds those in the problem did not significant improvement in Question 3 which requires the *division* operation but it started out small so this may not mean much. By contrast Error 5 which is that the number of terms in the resulting series is less than those in the problem showed improvement only in the *division* question because it occurred at a higher frequency in that question to start with.

Table 1: Number of Errors made by students classified according to error type.

Classified by Error type

	Interactive Tutorial Pretest No of errors of this type	Possible errors that students did not do	Interactive Tutorial Posttest No of errors of this type	Possible errors that students did not do	Fisher Exact Test results
Question 1-Multiplication					
E1	**15**	**6**	**6**	**15**	**0.006**
E3	**17**	**4**	**5**	**16**	**0.000**
E4	**17**	**4**	**6**	**15**	**0.001**
E6	**16**	**5**	**5**	**16**	**0.001**
Question 2-Power					
E1	**14**	**7**	**6**	**15**	**0.015**
E4	**14**	**7**	**8**	**13**	**0.061**
E6	**12**	**9**	**5**	**16**	**0.029**
Question 3-Division					
E1	**13**	**8**	**3**	**18**	**0.002**
E3	**12**	**9**	**3**	**18**	**0.009**
E4	**13**	**8**	**3**	**18**	**0.002**
E5	**13**	**8**	**2**	**19**	**0.000**

Table 2: Number of Errors made by students classified according to error type.

		Division	Power	Multiplication
Interactive Tutorial	**Pre**	**56**	**54**	**70**
Interactive Tutorial	Post	14	28	25
Tutorial + Mirror Modeler	**Pre**	**6**	**10**	**21**
Tutorial + Mirror Modeler	Post	0	17	1

COMPARISON TO PRIOR WORK

These results can also be compared to the results obtained in prior work (Alkhalifa and AlDallal, 2002) as is shown in the following table.

The Pretests in both experiments do not show significant differences in a Chi test which indicates that they can be compared to each other adequate replications. Additionally, the Pre and Post tests in the Interactive Tutorial case does not show any significant differences in the distribution which implies that students seem to learn all three tasks comparably, while significant differences do arise with a Chi value of 22.07 and $p < 0.000016$. This implies that the Mirror Modeler behaves in a way that is quite distinct from the way the Interactive Tutorial behaves and this should be visible clear because the Interactive Tutorial results in lower errors in all problem types while the Mirror Modeler results in higher error levels with series that have the *power* operation.

DISCUSSION

The Interactive education system shown here presents a highly influential method of teaching mathematical series such that it frees subjects from redundant calculations by presenting itself as a Cognitive Tool. The system allows students to alter critical variables and to watch the emerging series as a result of their changes and this seems to have positive influence on their level of learning. Improvements of 35.7%, 20.6% and 33.3% resulted and differences in error types are clearly related to the problem type. The individual student modeler is highly beneficial in the cases of the Division and Multiplication operations while it is counter productive with power operations. This clearly required further study but that is left for further work.

ACKNOWLEDGEMENTS

The author would like to thank first and foremost god for moments of insight that enlighten the darkest nights of confusion. The author would also like to thank Layla AlDallal for her strong support of the work presented here.

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