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Aggregated Maximum Entropy Variational Analysis Method for Magnetic Resonance Imagery

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ABSTRACT

There is a variety of computational paradigms for post-processing of biomedical magnetic resonance (MR) images based on the use of the maximum entropy (ME) image reconstruction method. Sometimes the ME-reconstructed image quality is insufficient to clinical analysis because of the degradations of many special features of the image, i.e., edges-stopping, localization of homogeneous zones, signal-to-noise ratios, textures degradation, etcetera. In this work, we propose to modify the conventional ME image reconstruction technique by aggregating it with the variational analysis method and address a new fused maximum-entropy-variational-analysis (MEVA) method for reconstruction and denoising of the MR images. Also, we propose an efficient computational scheme for numerical implementation of the MEVA algorithm using a Hopfield-type modified neural network, and demonstrate performance outcomes of the MEMA-reconstructed MR images.

I. INTRODUCTION AND BACKGROUND

Nowadays, the acquisition of biomedical images is one of the most important fashions in medical environment, and a great variety of visualization sensors exists, e.g. magnetic resonance (MR), tomography, x-rays, etc. In this study, we consider the MR images degraded by noise and blurring, and present a new fused maximum-entropy-variational-analysis (MEVA) method for their computational reconstruction. Also, we propose the neural network (NN) of the modified Hopfield-type structure [1] to implement the MEVA method computationally. We demonstrate through computer simulations the effectiveness of such the MEVA-NN algorithm for MR image enhancement.

Consider the linear mathematical model of a MR image degraded by the observation noise and the limitation of the resolution of a sensor system (blur operator) [6]

$$u(y) = \int_X A(x,y)v(x)dx + n(y) \qquad (1)$$

where, $v(x)$ is the objective biomedical image, $u(y)$ is the data measurements (the degraded MR image), $n(y)$ is the observation noise and $A(x,y)$ is the MR system point spread function (PSF). The lineal equation of observation (1) in a discreet form becomes

$$\mathbf{u} = \mathbf{Av} + \mathbf{n} \qquad (2)$$

where $\mathbf{v}$ Î $\{0, 1, \ldots, L-1\}^K$, L is the total number of image intensity levels, $\mathbf{v}$ is the original discrete image (representing the 256 tonality gray-scale pixel of intensity into 0 to 1 range), $\mathbf{n}$ represents the noise, and $\mathbf{u}$ is the observed discrete-form blurred noised image. In the problems related to the MR image reconstruction (enhancement), $\mathbf{A}$ represents a matrix-form approximated PSF. The initial image (vector $\mathbf{v}$) consists of a given number of the so-called *speckles* (or pixels v_k; $k = 1,\ldots, K$). Each of these speckles (pixels) is referred to as a portion of the power distribution in the observation domain. The nonlinear constraint on the image non-negativity (as the power is a non-negative function) is imposed by a set of inequalities

$$v_k \geq 0 \qquad \text{for all } k = 1,\ldots, K. \qquad (3)$$

Next, the constraint on the limit of the overall image energy (the total power) of the desired image is imposed via the calibration constraint [1]

$$\sum_{k=1}^{K} v_k = B. \qquad (4)$$

These two constraints (3) and (4) enable one to introduce the probabilistic interpretation of the *speckle* image model [1]. To proceed with such an interpretation, we consider the normalized image model

$$p_k = {v_k}/{B}; \quad k = 1,\ldots, K; \qquad \sum_{k=1}^{K} p_k = 1. \qquad (5)$$

II. THE FUSION METHODOLOGY

A. Image Entropy

The normalized non-negative values $\{p_k\}$ defined by (5) can be treated as the discrete probability distribution of the image that is characterized by the Shannon's *entropy*

$$H(p) = -\sum_{k=1}^{K} p_k \ln p_k = -a\sum_{k=1}^{K} v_k \ln v_k + b\,, \tag{6}$$

with the normalizing parameters, $a = 1/B$, $b = \ln B$. The ME criterion is indeed an important principle for modeling the prior probability density function $p(\mathbf{v})$ that has been used in the numerous processing applications. Following [1], we define here the image entropy function as

$$H(\mathbf{v}) = -\sum_{k=1}^{K} v_k \ln v_k\,. \tag{7}$$

B. Image Variational Functional

When performing the reconstructive MR image post-processing, the goal is to provide the noise reduction in the homogeneous domains with edge preservation. This can be formalized in the terms of the following variational optimization problem [3], [4], [5]

Minimize the image gradient flow $F(\mathbf{v}) = \int_{\Omega} g(|\nabla \mathbf{v}|)\, d\Omega$ (8)

where,

$$g(x) = \frac{1}{1 + \left(x/K \right)^2} \tag{9}$$

is the Perona-Malik approximation to the image gradient energy function [4] and Ñ**v** is the image gradient distribution over the scene W.

III. THE MEVA METHOD

We propose here the following fused image reconstruction strategy

$$\hat{\mathbf{v}}_{MEVM} = \arg\min_{\mathbf{v}} \left\{ \sum_{k=1}^{K} v_k \ln v_k + \frac{1}{2}\lambda_1 J_1(\mathbf{v}) + \frac{1}{2}\lambda_2 J_2(\mathbf{v}) + \lambda_3 \int_{\Omega} g(|\nabla \mathbf{v}|)\, d\Omega \right\} \tag{10}$$

where, »$_1$, »$_2$, and »$_3$ represent the regularization parameters,

$$J_1(\mathbf{v}) = \sum_{j=1}^{K} \left[u_j - \sum_{k=1}^{K} A_{jk} v_k \right]^2 \tag{11}$$

represents the objective function [1],

$$J_2(\mathbf{v}) = (\mathbf{L}\mathbf{v})^T \mathbf{L}\mathbf{v} = \mathbf{v}^T \mathbf{P} \mathbf{v} = \sum_{k=1}^{K} \sum_{i=1}^{K} P_{ki} v_k v_i \tag{12}$$

represents the Tikhonov's stabilizer [1], where $\mathbf{P} = \mathbf{L}^T \mathbf{L}$ and $\mathbf{L}$ is as a discrete-form approximation of the Laplacian operator [6].

Next, we propose to redefine the composed objective function in (10) as an energy function of the hypothetical modified Hopfield NN [1], [2] that leads to the following optimization problem for MR image reconstruction

$$\hat{\mathbf{v}}_{MEVM} = \arg\min_{\mathbf{v}} \left\{ E(\mathbf{v} \mid \boldsymbol{\lambda}) \right\}. \tag{13}$$

With these definitions, the expanded form for the NN energy function becomes

$$E(\mathbf{v} \mid \boldsymbol{\lambda}) = -\frac{1}{2} \sum_{k=1}^{K} \sum_{i=1}^{K} \left[-\lambda_1 \sum_{j=1}^{K} A_{jk} A_{ji} - \lambda_2 P_{ki} - 2\lambda_3 M_{ki} \right] v_k v_i - \sum_{k=1}^{K} v_k \left[-\ln v_k + \lambda_1 \sum_{j=1}^{K} A_{jk} u_j \right] + C_E \tag{14}$$

where, C_E is the nuisance shift that does not involve **v**.

IV. NN FOR IMPLEMENTATION OF THE MEVA METHOD

We propose to use the multi-state modified Hopfield-type NN developed in [1], [2] to implement the MEVA method. The outputs of the NN compose the output vector, $\mathbf{z}$ = sgn($\mathbf{W}\mathbf{v}$ + **è**), where **W** is the $K \times K$ matrix of the interconnection strengths of the NN, and **è** is the $K \times 1$ bias vector [1]. The output vector **z** is used to update the state vector **v** of the network: $\mathbf{v}'' = \mathbf{v}' + \Delta\mathbf{v}$ where, $\Delta\mathbf{v} = \Re(\mathbf{z})$ is a change of the state vector **v** that is computed by applying the state update rule $\Re(\mathbf{z})$ specified in [1]. The superscripts ″ and ′ correspond to the state values before and after network state updating, respectively. The energy function of such the NN is defined as [1]

$$E(\mathbf{v}) = -\frac{1}{2} \mathbf{v}^T \mathbf{W} \mathbf{v} - \boldsymbol{\theta}^T \mathbf{v}\,. \tag{15}$$

To find the NN-based solution to (10) with the energy function determined by (15) we specify the interconnection strengths W_{ki} and bias inputs θ_k of all the neurons of the NN as follows,

$$W_{ki} = -\lambda_1 \sum_{j=1}^{K} A_{jk} A_{ji} - \lambda_2 P_{ki} - 2\lambda_3 M_{ki} \tag{16}$$

$$\theta_k = -\ln v_k + \lambda_1 \sum_{j=1}^{K} A_{jk} u_j \;;\; k,\, i = 1, \ldots, K \tag{17}$$

that enable the NN to converge to the desired solution (10) at its stationary state [1].

V. SIMULATIONS

In this section, we report some computational simulations based on the proposed MEVA-NN algorithm for MR image reconstructive post-processing. In Figure 1, the observed 256-by-256 MR image (degraded and noised) is displayed. The reconstructed images are displayed in Figures 2 and 3 for the cases of applying the conventional ME method [1] and the proposed here fused MEVA method, respectively. The regularization parameters (l$_1$, l$_2$, and l$_3$) were adjusted using the methodology presented in [1] to gain the feasible improvement in the output signal-to-noise ratio (IOSNR). The reconstructions were performed for different combinations l$_1$, l$_2$, and l$_3$.. The resulting IOSNR are reported in Table 1. The images displayed in the Figures 2 and 3 were obtained using the combination of the regularization parameters from the last row of Table 1. These results manifest the feasible improvement in the image reconstruction achieved with the proposed MEVA method.

VI. CONCLUSIONS

The new approach to MR image reconstruction that fuses the maximum entropy and variational analysis methods into the aggregated MEVA technique was presented. The MEVA-NN algorithm demonstrated a feasible efficiency in enhancement/ reconstruction of the MR images if compared with the previously developed ME-NN technique. The fused MEVA-NN algorithm provides the enhanced localization of

Figure 1. Blurred and noised MR image.

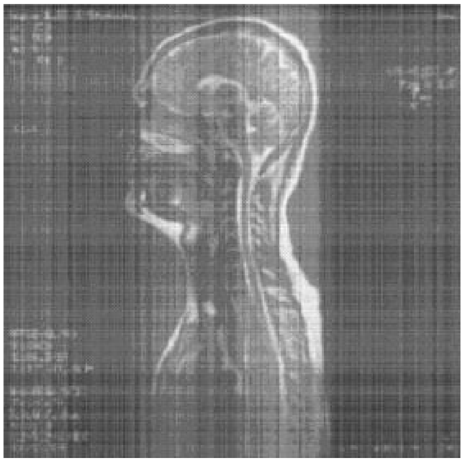


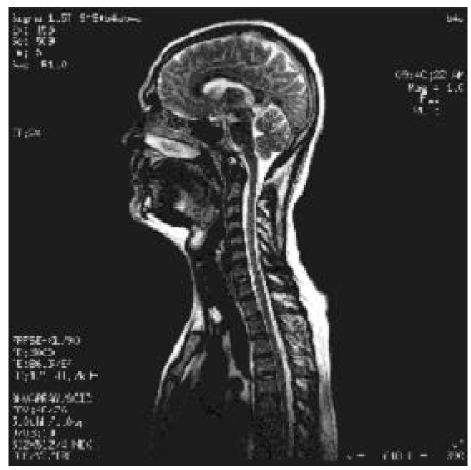
Figure 2. Image reconstructed using the ME-NN algorithm.

the homogeneous zones with better edge preservation. Also, the fused MEVA-NN algorithm admits the selection (adjustment) of the regularization parameters that provide the additional degrees of freedom of the overall image reconstruction procedure. The improvement in the reconstruction performance is quantitatively characterized by the IOSNR values reported in the Table 1.

Table 1. The values of the IOSNR and the resulting NN energy.

λ_1	λ_2	λ_3	ME-NN		MEVA-NN	
			IOSNR (dB)	E_{min}	IOSNR (dB)	E_{min}
7	4.5	1	-8.390843	2.882054	-2.898658	2.129680
8	5.5	1	-0.419754	1.936401	8.004568	1.601765
9	6.5	1	6.361728	1.669026	9.664276	1.437823
10	7.5	1	10.108438	1.310295	10.108438	1.174252
11	8.5	1	10.372660	1.205747	10.372660	1.099949
5.5	3.5	1	-7.153951	3.612093	-0.482031	2.314780
6.5	4.5	1	5.069690	2.036390	10.266486	1.602975
7.5	5.5	1	9.899443	1.417083	9.941760	1.216505
8.5	6.5	1	9.700362	1.192491	9.700362	1.061585
9.5	7.5	1	10.060438	1.036291	10.060438	0.944603

Figure 3. Image reconstructed using the fused MEVA-NN algorithm.



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