Chapter 10
Introducing Vagueness in the Mathematical Curriculum of Secondary Education: Experience in Greece

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ABSTRACT

Vagueness is a quality that is usually ignored or considered a deficiency for any system, in the broad sense of the word. However, vagueness seems to be a fundamental property of our world and so systems that are precisely defined are rather the exception than the norm (e.g., in most cases we ignore some details that are supposed not to affect the overall behavior of a system). Because of this "attitude", vagueness and its mathematics have not been introduced in secondary education; the goal is to change this. Therefore, the authors prepared a series of lessons that have been taught to middle school pupils and recorded the input we got from them. The authors report the structure of the lessons as well as the input received and the prospects of actually introducing vagueness in the mathematical curriculum of secondary education in Greece.

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INTRODUCTION

Science and technology progress every single day, sometimes with small steps, sometimes with big leaps. However, most of these new discoveries do not find their way into the curriculum of secondary education (primary education is something entirely different). The main obstacles to the introduction of "new" subjects into the curriculum are inertia and tradition. Teachers have learned a specific way to teach things and they always know what they must teach when they get into a classroom. By introducing "new" subjects, we break this circle and make the life of most educators difficult. Let us think about what happened when teachers had to learn how to use computers in their classes—most of them objected to the idea and considered computers as an unnecessary addition to the already heavy load of their work. Of course, today there are no classrooms without computers, but it took some years to achieve this. Inertia is the refusal to use anything new, refusal which will eventually make the life of teachers more difficult.[1] Fortunately, in our case the Ministry of Education allows teachers to experiment with innovative ideas and subjects and to examine how these can be best introduced into the current curriculum. In particular, teachers can create classes and/or sets of activities after the normal school hours. However, in our case, we managed to get permission from the school principal to make our "experiment" part of the normal school curriculum for one week.

Fuzzy set theory is a mathematical model of vagueness that was introduced by Lotfi Aliasker Zadeh (1965) [see (Syropoulos & Grammenos, 2020) for a modern overview of fuzzy mathematics]. Although fuzzy mathematics has found numerous applications in science and technology, there are still few texts that aim to introduce the theory to novice readers let alone high school students. Even though García-Honrado (2012) has argued in favor of the introduction of fuzzy sets into school education, to the best of our knowledge there are only two books attempting to teach fuzzy sets to novice people. In particular, (Mukaidono, 2001) is a book aiming to introduce fuzzy sets to novice readers while (Lin et al., 2018) is aiming to introduce fuzzy sets to high school students (the original book was written in Chinese obviously for Chinese high school students).

Intuitionistic fuzzy sets (Atanassov, 1986, 2012) and neutrosophic sets (Smarandache, 2006) are two extensions of fuzzy sets that "violate" the idea that if an element x has membership degree equal to d, then its complement has membership degree equal to $1–d$. Clearly, these ideas are quite interesting, but in order to introduce them one has to first introduce fuzzy sets and before them one has to introduce the notion of vagueness.

We have designed a mini course that introduces, in this order, vagueness, ordinary sets and their basic operations, fuzzy sets and their basic operations, and, finally, intuitionistic and neutrosophic sets. This mini course is based on our previous experience exposing fuzzy sets to adults and taking into consideration the special needs of our students. The school where we work (i.e., the 2nd Gymnasium of Xanthi, Greece) is a typical middle school. Each week students are taught 4 non-consecutive lessons in mathematics, each of which lasts 50 minutes. Thus, the mini course consisted of 4 lessons and replaced the normal teaching of mathematics for a week. The course was taught to two different classes, each having roughly 25 students.