Chapter 27
Distributional Uncertainty for Spectral Risk Measures

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ABSTRACT

Spectral risk measures, primarily introduced as an extension for expected shortfall, constitute a prominent class of risk measures that take account of the decision-makersrisk-aversion. In practice, risk measures are often estimated from data distributions, and due to the uncertain character of the financial market, one has no specific criterium to pick the appropriate distribution. Therefore, risk assessment under different possible scenarios (such as financial crises or outbreaks) is a source of uncertainty that may lead to concerning financial losses. The chapter addresses this issue, first, by adapting a robust framework for spectral risk measures, by considering the whole set of possible scenarios instead of making a specific choice. Second, the author proposes a variability-type approach as an alternative for quantifying uncertainty, since measuring uncertainty provides us with information about how far our risk measurement process could be impacted by uncertainty. Furthermore, the stated theory is illustrated with a practical example from the NASDAQ index.

INTRODUCTION

In risk management, risk measures are used to calculate required capital that acts as a hedging against the inherent risks. Over the last few decades, numerous risk measures have been introduced. Typical examples, in the financial literature, are Value-at-Risk, Expected Shortfall and the much broader classes of coherent and convex risk measures introduced, respectively, in Artzner et al. (1999) and Föllmer and Schied (2002) for which the theoretical definition is founded upon a set of reasonable properties that a measure of risk may satisfy. (For more details, the reader is referred to Pflug and Römisch, 2007, Delbaen, 2012, McNeil et al., 2005, and Föllmer and Schied, 2016).

Among coherent risk measures the law-invariant and co-monotonic additive ones co-incide with spectral risk measures (Kusuoka, 2001). They were introduced by Acerbi (2002) and are arguably con-

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sidered as the most important extensions of Expected Shortfall. Moreover, since decision-makers typically differ on how they perceive the notion of risk, spectral risk measures are designed to incorporate such discrepancy. Hence, beyond fulfilling most of the suitable theoretical properties of a reasonable risk measure, a spectral risk measure is characterized with a weighting function φ that account for the psychological attitude of different profiles, by reflecting the subjective risk-aversion. This full range of attractive features makes the class of spectral risk measures of practical interest and justifies the framework of the present chapter.

In particular, law-invariance is an important feature in practice since it is a necessary property for a risk measure to be estimable from empirical data. In this way, it is totally legitimate to argue that the most suitable risk measures, in practice, are the law-invariant (distribution-based) ones (see e.g., Henryk and Silvia, 2006, Adam et al., 2008, Emmer et al., 2015, and Wang et al., 2018). That is to say, for risk measurement practitioners use risk measures based on a given probability measure according to different alternative scenarios. The author considers that each probability measure reflects the underlying economic situations (such as: normal situation, volatile situation, crash, outbreak, financial crisis). However, improper distribution assumptions can lead to wrong decisions and then to significant financial losses, some examples of real incidences are reported in Dowd (2007, Chap 16), Bloom (2014), Wang et al. (2017) and Jokhadze and Schmidt (2018). Therefore, the choice of probability measures constitutes a source of uncertainty in the risk management process. This situation naturally requires considering robust risk measures, meaning risk measures that are insensitive to the choice of probability measures.

This uncertainty regarding the choice of the proper probability measure (then the proper risk measure) has motivated the investigation of two issues:

- How to overcome uncertainty?
- How to measure uncertainty?

The present chapter contributes to the existent literature recently addressing this issue. In this context, Wang and Ziegel (2018) propose a robust framework based on the worst- case, especially for Value-at-risk (VaR) and Expected Shortfall (ES), as a theoretical foundation for the recent Basel regulations. Jokhadze and Schmidt (2018) and Righi (2018b) explore, additionally to the worst-case approach, results for the weighting average as a construction for robust risk measures and study some issues such as financial properties and dual representations. Works such those of Pflug et al. (2012) and Guo and Xu (2019) focus on the portfolio optimization issue under uncertainty regarding probability. From another perspective and instead of proposing robust risk measures, works such Shapiro (2017) consider robust stochastic optimization under a set of uncertainty, referring to a set of probability measures. The author intends to address the above formulated questions by adapting a consistent framework to the class of spectral risk measures. Moreover, the author intends to propose an alternative approach for measuring uncertainty itself. Since quantifying uncertainty provides us with information about to what extent our risk measurement process could be impacted by uncertainty and it may even be seen as a penalization to add to the required capital for covering the held financial position. To measure uncertainty, Jokhadze and Schmidt (2018) proposed a superposed measure that evaluates the dispersion, of a collection of risk measure, relatively to some reference risk measure ρ0. At this point, the approach suggested by the authors is interesting except that it stays dependent on the choice of a risk measure of reference. In this sense, the author is proposing a variability-type approach, especially since the monetary risk mea-

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