Chapter 7
Numerical Methods for Solving Dynamic Equilibrium Equations

ABSTRACT

Once the number of degrees of freedom exceeds a certain number, it would be impossible to solve the dynamic equilibrium equation manually, hence the need to switch to a numerical resolution, whose general principle is to convert a dynamic equation into a static one. We are interested, for the dynamic analysis of the structures and the continuous media, in "one-step" algorithms rather than "multi-step" one. It is mainly because the systems to be solved are of large size and that it is important to minimize the number of operations and value to be memorized to the detriment, if necessary, of precision. A "one-step" algorithm, like that of Newmark, makes it possible to calculate the solution at time t_{n+1}*, starting from the solution at time* t_n*. In addition to the disadvantage of requiring the storage of several steps, the "multi-step" algorithms such as that of Houbolt requires a startup procedure. This chapter allows the reader to enumerate and understand different numerical method with different examples.*

INTRODUCTION

This chapter describes the different numerical resolution method of dynamic equilibrium equation, which all have the same principle, which is the conversion of a dynamic equation into a static one.

Most of the known dynamics methods, the frequency domain analysis, the Duhamel integral, presuppose the validity of the superposition principle and therefore do not apply to non-linear problems.

Direct integration, step by step with a constant or variable time step Dt, evaluates the response during the time step from the initial conditions to the beginning of the time step U_0 ,$\dot{U}_0$ and the evolution of the load during the step.

The response at each time step is an independent analysis that does not require the validity of the superposition principle. Step by step integration methods provide a general approach to the problem of linear and non-linear dynamics

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Before talking about numerical methods, we allow ourselves to give a brief summary in the form of a table on what has been done so far for the resolution of one degree of freedom system under different loads types.

DIFFERENT STRUCTURAL RESPONSES OVERVIEW

The table below gives an overview of different responses of the structure corresponding to the different types of solicitations; these later can be enumerated as follows: harmonic, periodic, impulsive and any type of solicitations.

For Duhamel integral there are limitations, which are:

- The assumption of the linear system
- The solutions are not always possible example earthquake
- The integral is not generalized; for each load there is a separate solution that can't be scaled.

In order to overcome these limitations, numerical methods are used which offer several advantages, namely:

- The use of these methods in software (seismic simulation)
- Treatment of non-linear systems
- Possibility of generalizing these methods
- Programmability of methods

EXACT METHOD BY INCREMENTS

This method consists in dividing the history of the charge into intervals, generally constant, and in admitting that the charge varies linearly over the interval. There is therefore a linear approximation of the load per interval, and the response for the linearized load is evaluated for a continuously variable load, the solution is improved by reducing the time step h = Dt (constant or variable).

Considering a step Dt = t1-t0 and setting t = t- t0, we then have the load:

$$P(\tau) = P0 + \alpha\tau \quad (1)$$

The equation of motion becomes:

$$M\ddot{U} + kU = P_0 + \alpha\tau \quad (2)$$

The response comprises a free vibration component U_h and the particular solution U_p corresponding to the load:

$$U(\tau) = Uh_{(}\tau) + UP(_{\tau}) \quad (3)$$

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