Chapter 6
Extracting Eigenvalues and Eigenvectors Technique

ABSTRACT

In the process of solving dynamics problems at a given stage, one needs to have the dynamic characteristic of the structure in order to be able to push the calculation (resolution). Further, these characteristics are the eigenvalues and eigenvectors. In this chapter, some important points are dealt with, namely problem with standard eigenvalues, property of eigenvalues and eigenvectors, Rayleigh quotient, offset of eigenvalues, and different methods for extracting eigenvalues and eigenvectors are presented. At the end, some examples are exposed.

INTRODUCTION

This chapter describes the different methods of extracting eigenvalues and eigenvectors as well as their classifications and their characteristics. Calculating the eigenvalues and the eigenvectors of matrices is one of the most important problems in linear numerical analysis. Techniques requiring knowledge of the spectrum of matrices are used in fields as varied as quantum mechanics, structural analysis. Knowledge of the vectors and eigenvalues provides key information on the linear application considered. There are moreover many cases where this knowledge totally characterizes the linear application. The naive method of ðnding the eigenvalues of a matrix involves ðnding the roots of the characteristic polynomial of the matrix.

BRIEF REMINDER

The equation of motion, case of free vibrations for example (without load term) is written

$$\left[M\right]\left\{\ddot{U}(t)\right\}+\left[K\right]\left\{U(t)\right\}=0 \tag{1}$$

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There is a solution of the form

$$U(t) = \phi \sin(\omega t + \theta) \quad (2)$$

$$\ddot{U}(t) = -\varphi\,\omega^2 \sin(\omega t + \theta)$$

with ϕ is a vector, ω the vibration frequency, θ the phase angle.

By replacing $U(t)$ and $\ddot{U}(t)$ in equation (1) we obtain

$$([K] - \omega 2^{[}M]) \phi \sin(\omega t + \theta) = 0 \quad (3)$$

Which must be verified whatever time t, hence the condition

$$([K] - \lambda[M]) \phi = 0 \quad (4)$$

Because $\sin(\omega t + \theta) \neq 0$

Equation (4) defines a eigenvalues problem, where [M] is a defined-positive mass matrix, $\lambda_i = \omega_i^2$ and ωi Ar_e the eigen frequencies expressed in rad / s.

The condition for existence of a non-trivial solution of equation (4) is:

$$P(\lambda) = \det([K] - \lambda[M]) = 0 \quad (5)$$

Where P (λ) is the characteristic polynomial of order n with n is the number of degrees of freedom.

Note

1. If the matrix [K] and [M] are positive, the eigenvalues are real and positive
2. If the matrix [M] is singular, there are infinite eigenvalues.
3. If the matrix [K] is singular, it exits zero eigenvalues.

STANDARD EIGENVALUES PROBLEM

The problem with the standard eigenvalues is defined by the equation:

$$(A - \lambda I) \phi = 0 \quad (6)$$

Where I is the identity matrix, ϕ is an eigenvector of A, λ is an eigenvalue of A and equation (6) expresses that by multiplying a matrix by one of its eigenvectors we obtain n proportional vectors.

If A = [K]e (the stiffness matrix of a finite element), equation (6) will define the Eigen frequencies of the element. The solution of the whole is written as follows:

$$[K]^e[\Phi] = [\Phi]\Lambda \quad (7)$$

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