Clique Finder:
A Self-Adaptive Simulated Annealing Algorithm for the Maximum Clique Problem

Sarab Almuhaideb, Department of Computer Science, College of Computer and Information Sciences, King Saud University, Saudi Arabia

Najwa Altwaijry, Department of Computer Science, College of Computer and Information Sciences, King Saud University, Saudi Arabia

https://orcid.org/0000-0002-7386-1886

Shahad AlMansour, Department of Computer Science, College of Computer and Information Sciences, King Saud University, Saudi Arabia

Ashwaq AlMklafi, Department of Computer Science, College of Computer and Information Sciences, King Saud University, Saudi Arabia

AlBandery Khalid AlMojel, Department of Computer Science, College of Computer and Information Sciences, King Saud University, Saudi Arabia

Bushra AlQahtani, Department of Computer Science, College of Computer and Information Sciences, Saudi Arabia

Moshail AlHarran, Department of Computer Science, College of Computer and Information Sciences, King Saud University, Saudi Arabia

ABSTRACT

The maximum clique problem (MCP) is a classical NP-hard problem that has gained considerable attention due to its numerous real-world applications and theoretical complexity. It is inherently computationally complex, and so exact methods may require prohibitive computing time. Nature-inspired meta-heuristics have proven their utility in solving many NP-hard problems. In this research, the authors propose a simulated annealing-based algorithm that they call clique finder algorithm to solve the MCP. The algorithm uses a logarithmic cooling schedule and two moves that are selected in an adaptive manner. The objective (error) function is the total number of missing links in the clique, which is to be minimized. The proposed algorithm was evaluated using benchmark graphs from the open-source library DIMACS, and results show that the proposed algorithm had a high success rate.

KEYWORDS

Cooling Schedule, DIMACS, Maximum Clique Problem, Metaheuristics, Nature-Inspired Methods, NP-Hard Problem, Self-Adaptive, Simulated Annealing

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INTRODUCTION

The Maximum Clique Problem (MCP) is a classical NP-hard combinatorial optimization problem concerned with finding the largest subset in a graph, where all nodes in the subset share an edge. Given an undirected graph $G\left(V,E\right)$, a clique C is a subset of the graph, such that there is an edge between any two vertices in C. A clique C is said to be maximal if it is not a subset of any larger clique of G. The clique in G with the largest cardinality is known as a maximum clique, i.e. it cannot be extended to a larger one. The MCP seeks a maximum clique in a graph. A related problem is the maximum weighted clique problem: an NP-complete problem to find the clique with the maximum weight sum in a given undirected graph.

Definition 1: ***Maximum Clique Problem (MCP).*** *Let $G\left(V,E\right)$ be an undirected graph with vertex set $V=\left\{1,2,\ldots,n\right\}$ and edge set $E\subseteq V\times V$. A clique C is a subset of V such that every two vertices in C are adjacent; $\forall\, u,v \in C;\left(u,v\right) \in E$.*

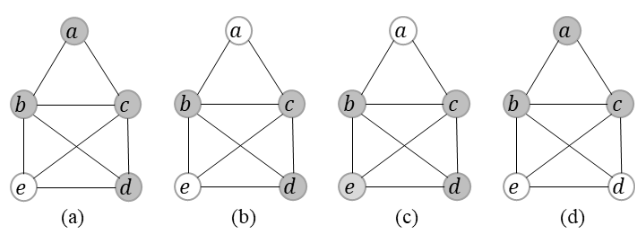
The MCP is an optimization problem of size $n=\left|V\right|$, the number of vertices in G. MCP is NP-hard because the complexity of this problem is the in order of 2^n (Wu & Hao, 2015).

Figure 1 presents an illustration of MCP. The shaded vertex set in Figure 1.a does not represent a clique, because vertices $\left(a,d\right)$ are not directly connected to each other. The second vertex set in Figure 1.b $(b,c$ and $d)$, forms a clique because all shaded vertices are connected to each other. However, there is a vertex in the graph, e, that is connected to all the set vertices, so this set is not a maximal clique. The vertex set in Figure 1.c $\left(b,c,d,e\right)$ is a maximal clique with cardinality equal to 4, because all vertices are connected to each other and vertex $\left(a\right)$ is not connected to every vertex in Figure 1.c. The vertex set $\left(a,b,c\right)$ in Figure 1.d is another maximal clique with cardinality 3. The clique in Figure 1.c is thus considered a maximum clique (Vilakone, Park, Xinchang, & Hao, 2018).

Two problems equivalent to the MCP are the maximum independent set problem (MIS) and the minimum vertex cover problem (MVC). MIS problem is a problem which seeks the largest set of vertices that are not related to each other. Consider a graph $G=\left(V,E\right)$ with vertex set V and edge set E and its complement $\bar{G}=\left(V,\bar{E}\right)$ where $\bar{E}=\left\{\left(v,w\right)\notin E;v,w\in V,v\neq w\right\}$. An independent set is a vertex set where each pair elements are not adjacent. An independent set is maximal when it is not a subset of any larger independent set and maximum when there are no larger independent sets in the graph. This problem is similar to MCP in getting a maximum independent set I of G provided that I is a clique of $\bar{G}$.

The minimum vertex cover generates the smallest subset of the vertex set, where vertices in the subset cover all edges (Chen, Kou, & Cui, 2016). Given an undirected graph $G=\left(V,E\right)$, the minimum

Figure 1. Relationship between maximal and maximum clique



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