Chapter 8
Advances in Fuzzy Dynamic Programming

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ABSTRACT

This chapter considers the use of fuzzy dual numbers to model and solve through dynamic programming process mathematical programming problems where uncertainty is present in the parameters of the objective function or of the associated constraints. It is only supposed that the values of the uncertain parameters remain in known real intervals and can be modelled with fuzzy dual numbers. The interest of adopting the fuzzy dual formalism to implement the sequential decision-making process of dynamic programming is discussed and compared with early fuzzy dynamic programming. Here, the comparison between two alternatives is made considering not only the cumulative performance but also the cumulative risk associated with previous steps in the dynamic process, displaying the traceability of the solution under construction as it is effectively the case with the classical deterministic dynamic programming process. The proposed approach is illustrated in the case of a long-term airport investment planning problem.

DOI: 10.4018/978-1-7998-5357-2.ch008

INTRODUCTION

While deterministic optimization problems are formulated with known parameters, very often real-world problems include unknown parameters (Delgado et al., 1987). When the parameters are only known to remain within given bounds, one way to tackle such problems is through robust optimization (Ben-Tal et al., 2009). When probability distributions are available for their values, stochastic optimization techniques (Ruszczynski et al., 2003) may be used to provide the most expected feasible solution. An intermediate approach adopting the fuzzy formalism to represent the parameter uncertainties has been also developed (Zimmermann, 1986). These three approaches lead in general to cumbersome computations. Also, in many situations the optimal solution cannot be applied exactly according to implementation constraints which have not been considered explicitly in the formulation of the problem. In that case post optimization sensibility analysis (Gal et al., 1997) resulting often in an important computational effort must be performed.

In this chapter, the fuzzy dual formalism is proposed to treat parameter uncertainty and solution diversion in mathematical optimization problems. This formalism adopting a simplified version of fuzzy numbers provides feasible solution approaches with respect to the resulting computational needs. The case of dynamic programming is more particularly considered in this chapter. Since its development in the mid-fifties by Richard Bellman, Dynamic Programming has become one of the main tool to cope with the optimization of dynamical systems and sequential decision problems, either in the control field or in the Operations Research field. Fuzzy Set Theory has been developed by Zadeh in the early sixties and has established itself as an important mathematical tool to address uncertainties and imprecision in treating real world problems. So, as soon as 1970, Bellman and Zadeh put together these concepts to propose a sequential decision tool under uncertainty leading to what has been called *fuzzy dynamic programming*. Although many decision problems are both sequential and uncertain, it has to be recognized that this promising technique has not encountered the expected popularity. So in this chapter, fuzzy dynamic programming with fuzzy dual numbers, i.e. Fuzzy Dual Programming is introduced and a case study is produced. At first, the general approach proposed by Bellman and Zadeh, leading to the fuzzy optimality principle, will be stated. Then different settings of fuzzy dynamic programming problems in conjunction with the corresponding solution approaches have been introduced, while more recent contributions in that field (in general Kacprzyk and Esogbue) will be discussed. Then the main limitations and difficulties common to these approaches will be outlined and analyzed. Beyond the computational burden generated by these approaches, the traceability of the proposed results is extremely poor. These have been the main negative factors for the adoption of Fuzzy Dynamic Programming techniques. Here, to improve both aspects, conceptual questions regarding first the equal logical treatment applied to constraints and performance indexes, and then regarding the fuzzy comparison process of different partial performance indexes, will be analyzed and directions for improvement will be proposed. With respect to the first point, either or not the optimization problem has a fuzzy character, only feasible solutions are of interest for the decision maker, so a first step before comparing the performances of solutions is to assess in a clear and independent way their feasibility, even when it is assessed in fuzzy terms. Then the current approach where performance levels and feasibility are merely merged using a logical *and* is considered questionable, so a new logical operator will be discussed. The second point is central in the dynamic programming process since it allows not only to progress towards the whole solution but also to weed out partial non optimal solutions and lessen the remaining computational burden. So to make effective the fuzzy dynamical programming process, the use of a class of fuzzy numbers easily compa-

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