Chapter 7
Applications of the Elementary Theory of Catastrophes in Aviation

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ABSTRACT

In this chapter, the author considers the connection between the mathematical theory of catastrophes and certain characteristics of the aviation system. The basis for this connection is the similarity of the concept of "catastrophe" in aviation and mathematics. With small perturbations of some systems, their state changes little. At small perturbations of other systems, a sharp transition occurs to another state. This transition is called a jump or a catastrophe. A catastrophe can also be called a loss of stability. Comparison of the notion of "stability" in aviation and mathematics is part of this study. The mathematical representation of the simplest aviation systems and their disasters is shown in elementary examples. It does not use a deep mathematical apparatus as well as special studies of aviation systems. The aim of the work is to direct the aviation specialist's eye to the mathematical theory of catastrophes, and to direct the mathematician's attention to the problems of aviation safety.

BACKGROUND

The mathematical theory of catastrophes has been accepted into analytics' standard toolbox in ship stability, fluid geometry, optics, thermodynamics, and other applications. The problem is that among these applications there is practically no research of automated systems in aviation. It is known that any important aviation system must have a serious mathematical apparatus. Therefore, the study of its functioning should contain mathematical methods. But aviation specialists, as a rule, do not have deep mathematical knowledge. As a rule, mathematicians are not familiar with the basics of the functioning of aviation systems. Therefore, it makes sense to consider the proposed theory. One of its scientific fields can be the analysis of air disasters or in other words aviation catastrophes.

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The air disaster or catastrophe is an event that occurred during the flight operation of an aircraft. This event leads to severe consequences for persons associated with exploitation, and destruction or damage to the aircraft itself. In other words, the air disaster is a significant change in the state of people and aircraft in the process of their standard functioning (Yakunina, 2017). In this sense, the definitions of the catastrophe are similar in aviation and in the mathematical theory of catastrophes. Catastrophes in mathematics are called spasmodic changes in the state of a mathematical system that arise as a sudden response of the system to a smooth change in external conditions.

First of all, we made an attempt to show the relationship between the concept of "stability" in aviation (Pashkovsky, 2018). and mathematics. We also showed the simplest examples of the application in aviation of the elementary concepts of catastrophe theory. At the same time, we do not penetrate deeply into any of these interesting areas of human activity. The aim of the work is to acquaint specialists in the field of aviation systems with some concepts of the mathematical theory of catastrophes. At the same time, we want to acquaint researchers of the theory of catastrophes with the problems of aviation safety.

The theory of catastrophes, as a separate section of research, appeared in the second half of the 20th century. Its sources were (Arnold, 1990, Golubitsky & Guillemin, 1973, Milnor, 1963) the theory of singularities of smooth mappings and the theory of bifurcations of dynamical systems. The theory of singularities of smooth mappings is a generalization of the investigation of functions to a maximum and a minimum. The theory of singularities of smooth mappings has found applications in the theory of stability of motion of dynamical systems, the theory of equilibrium position bifurcations, geometric and wave optics. Then this theory, along with the applications, was called the theory of catastrophes.

Among the works on catastrophe theory (Poston & Stewart, 1978, Brocker & Lander, 1975, Zeeman, 1976) there are studies of ship stability, fluid geometry, optics, thermodynamics, modeling of brain activity and so on.

Terminology and Scheme of Application of the Theory of Catastrophes

As in any new theory, the mathematical theory of catastrophes has not yet established a single terminology. It is important that as many aviation experts as possible get to know this theory. Therefore, it makes sense to bring here some competing terminology systems (Poston & Stewart, 1978). Their terms are used by researchers in applications, depending on various needs.

In catastrophes theory, the mapping is considered:

$$f : R^n \times R^r \rightarrow R$$

Elements of this mapping can be called (in various combinations) according to the terminology table 1. Other terms are sometimes used.

The scheme of most applications of the theory of catastrophes is as follows (Arnold, 1990):

- The process being studied is described with the help of a number of control and internal parameters.
- The states of equilibrium of the process form the surface of a particular number of dimensions in this space.
- The projection of the surface of equilibrium on the plane of the control parameters can have singularities that are assumed to be singularities of general position.